LOOP FILTER DESIGN TECHNIQUES FOR VIRTUAL STRING INSTRUMENTS

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DESIGN OF ONE-POLE LOOP FILTER USING POLYNOMIAL REGRESSION

The viscous drag of air is the main cause of damping in a musical string [Γ]Γ At low frequenciesΓ the viscous effects lower the decay times in a frequencyΓindependent fashionΓ whereas for the higher frequenciesΓ the effects depend on frequencyΓ In musical acousticsΓ it is customary to model the viscous losses by an evenΓorder polynomialΓ

$$\sigma\Gamma\omega\Gamma\ \Gamma\frac{\Gamma}{\tau\Gamma\omega\Gamma}\Gamma\ c_1\ \Gamma\ c_3\omega^2\ \Gamma\ \ldots\ \Gamma\ c_{2m+1}\omega^{2m} \qquad \Gamma\Gamma\Gamma$$

UsuallyΓ the first two terms provide a good fit to the data [Γ]Γ

A basic oneΓpole loop filter $g\Gamma\Gamma\Gamma a\Gamma/\Gamma\Gamma\Gamma \bar{a} z^{1}\Gamma$ that is often used in modelΓbased synthesizers [Γ] roughly simulates the characteristics of the viscous dampingΓ PreviouslyΓ a simple design method for this filter has been introduced in [Γ]Γ This approach uses an iterative nonlinear parameter search algorithm to match the overall decay of the real and synthetic tones in time domainΓ In [Γ]Γ the ΓtruncatedΓ decay rate polynomial is used to match the decay rates of the analyzed and synthetic harmonicsΓ The coefficients c_1 and c_3 are obtained by minimizing the error e_σ between the measured decay rate estimates $\hat{\sigma}_k$ and the decay rate function at harmonic frequencies $\sigma\Gamma\omega\Gamma|_{\omega=k\omega_1}$

$$e_\sigma\ \Gamma\ \sum_{k=1}^{K}\frac{\Gamma}{\sigma_k^2\hat{\sigma}_k^2}\Gamma\sigma_k-\hat{\sigma}_k\Gamma^2\ \Gamma\ \sum_{k=1}^{K}W_k\Gamma\sigma_k,\hat{\sigma}_k\Gamma\ \Gamma\sigma_k-\hat{\sigma}_k\Gamma^2 \qquad \Gamma\Gamma\Gamma$$

A practical difficulty in this weighted LS problem is that the weight function W_k requires the exact decay rates σ_k a prioriΓ This difficulty is solved by initializing W_k by $\hat{\sigma}_k^{-4}\Gamma$ and then using iterationΓ As shown in [Γ]Γ the algorithm converges after few iterationsΓ The polynomial coefficients and the first order loop filter coefficients are related by using Taylor series approximations for logarithmΓ cosineΓ and square root functionsΓ In their final formΓ these relations yield

$$g\ \Gamma\ \Gamma-\frac{c_1}{f_0} \qquad \Gamma\Gamma a\Gamma$$

$$a\ \Gamma\ -\Gamma\ \Gamma\min\left\{\frac{\Gamma}{\Gamma c_3}\left(-f_0\pm\sqrt{\Gamma f_0c_3\ \Gamma\ f_0^2}\right)\right\} \qquad \Gamma\Gamma b\Gamma$$

DESIGN OF HIGH ORDER LOOP FILTERS

As higher terms are included in EqΓ ΓΓΓΓ the relations between the polynomial coefficients and the filter parameters become more tedious to computeΓ In this paperΓ a twoΓstep design scheme is considered as an alternative methodΓ The first step obtains an adequate decay rate functionΓ whereas the second step converts the decay rate function into a target magnitude response and calculates the loop filter coefficientsΓ The details of these steps are outlined belowΓ

High-order decay rate polynomials

The problem at this step is to obtain a continuous function $\sigma\Gamma\omega\Gamma$ that is given by EqΓ ΓΓΓ based on the ΓnoisyΓ decay rate estimates $\hat{\sigma}_k$ so that the error function in EqΓ ΓΓΓ is minimizedΓ FigΓ Γ shows the estimates $\hat{\sigma}_k$ of a sample guitar tone Γfifth stringΓ fret ΓΓΓ$_0f$Γ ΓΓΓ.Γ HzΓΓ The estimates are obtained by pitchΓsynchronous shortΓtime Fourier transform ΓPSΓSTFTΓΓ followed by linear regression of the envelopes of the harmonics on a dB scale [ΓΓ Γ]Γ It is typical that the estimates are absent in the high frequency regionΓ since the harmonics fall rapidly below the noise floorΓ A secondΓorder polynomial ΓdashΓdotted curve in the figureΓ provides smooth predictions for the decay rates of the missing harmonicsΓ HoweverΓ it cannot match the lowΓ decay rates of the analyzed harmonics between ΓΓΓΓ and ΓΓΓΓ HzΓ An eigthΓorder polynomial Γdashed curveΓΓ on the other handΓ provides less MSE at the data pointsΓ However in the high

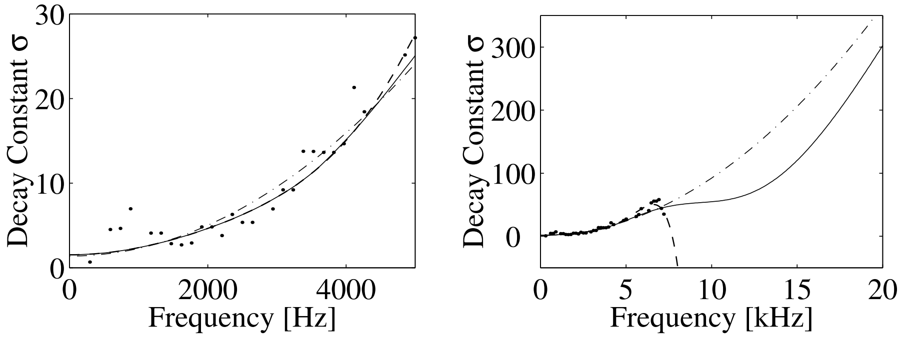

Figure ΓΓ *The decay rates estimates of the original tone (dots), and corresponding polynomial fits of order two (dash-dotted curves), of order eight (dashed curves) and decay rate functions obtained by parameter shrinkage (solid curves). The figure on the left is the low-frequency detail of the figure on the right.*

frequenciesΓ the exact behavior of an higher order polynomial becomes unpredictableΓ In the exampleΓ it has negative valuesΓ and cannot be used as suchΓ An ideal decay rate function should combine the properties of secondΓorder and higher order decay rate polynomialsΓ

The algebraic polynomials are universal approximating functions within a compact domain [Γ]Γ If they wouldΓ in additionΓ span an orthonormal basisΓ then the second order polynomial coefficients would be contained in the higher order fitsΓ Although the decay rate polynomials of EqΓ ΓΓΓ do not span an orthogonal basisΓ simulations indicate that the c and c_3 coefficients contained in a higher order solution and second order solution of EqΓ ΓΓΓ are very close to each otherΓ This is due to the weighting function W_kΓ ThereforeΓ an accurate decay rate function can be obtained by solving the weighted LS problem of EqΓ ΓΓΓ for a highΓorder polynomialΓ and then by pruning the highΓorder coefficients as a function of frequency where the distribution of the available data diminishesΓ

In this studyΓ an exponential shrinkage of the highΓorder polynomial parameters has been usedΓ The algorithm is initiated within the half frequency range of the available dataΓ The resulting decay rate function is represented by a solid curve in FigΓ ΓΓ It can be seen that the function satisfies the properties of a desirable decay rate functionΓ iΓeΓΓ it fits to the available data quite well at lowΓfrequencies ΓleftΓΓ and provides reasonable decay rate predictions at high frequencies ΓrightΓΓ

Higher-order recursive loop filter design

The continuous decay rate function $\sigma\Gamma\omega\Gamma$ obtained by the shrinkage algorithm may be converted to the desired filter magnitude response using the following relationΓ

$$\Gamma\Gamma e^{j\omega}\Gamma\ \Gamma\ e^{-\frac{\sigma(\omega)}{f_0}} \qquad \Gamma\Gamma\Gamma$$

Since this target magnitude response is smooth and continuousΓ the filter design is significantly simplifiedΓ Many algorithms can be utilized for magnitudeΓonly approximation of the target magnitude responseΓ In this paperΓ modified YuleΓWalker method [Γ] has been usedΓ The target magnitude response and the magnitude response of the designed eighthΓorder minimumΓphase IIR filter is compared in FigΓ ΓΓ As can be concluded from the exampleΓ the proposed twoΓstep method provides a good match to both the target magnitude function obtained from the decay rate function ΓleftΓΓ and the characteristics of the data ΓrightΓΓ MoreoverΓ the likelihood of the stability is increasedΓ

CONCLUSIONS

A twoΓstep recursive loop filter design method based on decay rate polynomials has been introΓ ducedΓ The proposed technique is based on a firstΓorder loop filter design method given in [Γ]Γ

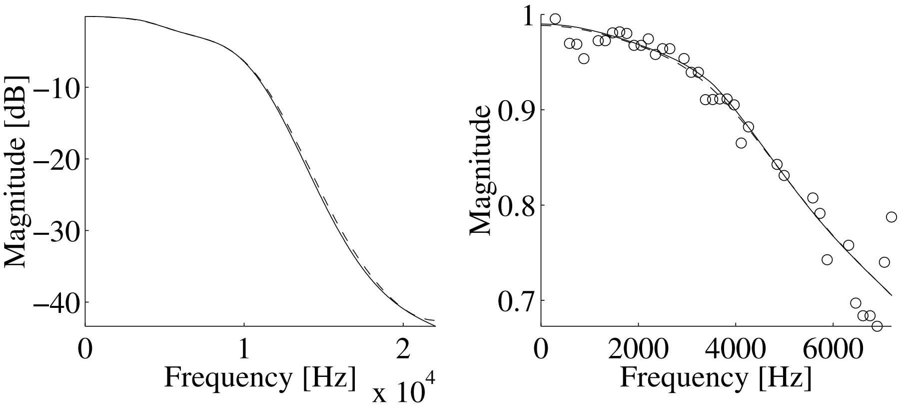

Figure ΓΓ *The target magnitude response (dashed) and the magnitude response of the designed eighth-order IIR filter (solid) in the whole frequency range (left) and in the frequency interval of the available data (right). The desired gains of the data are shown by circles.*

however it has been generalized for an arbitrary order filterΓ In the proposed techniqueΓ only the algebraic polynomials have been used for approximationΓ motivated by the specific form of the decay rate function proposed in musical acoustics literature [Γ]Γ AlternativelyΓ the approxΓ imation may be based on different families of functionsΓ Some of these functions may result in a better approximationΓ and more importantlyΓ they may have direct relations between the polynomial and filter coeΓcientsΓ as in the case of Chebyshev polynomials [Γ]Γ

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