DIPLOMA THESIS

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THEORETICAL AND PRACTICAL STUDIES ON THE BEHAVIOUR OF ELECTRIC GUITAR PICK-UPS

A Project in the
Department of Electrical Engineering
Acoustics Laboratory
at the
Helsinki University of Technology
by
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Preface

The diploma thesis is the final part of my studies in electrical engineering at the Fachhochschule Kempten. I received the offer to take part in the exchange program between the Helsinki University of Technology and the Fachhochschule Kempten. Here I want to express my gratitude to Professor Dr. Johannes Steinbrunn and Professor Dr. Tapani Jokinen for establishing this partnership and encouraging students to participate in it.

I want to express my special thanks to my supervisor Professor Dr. Matti Karjalainen from the Helsinki University of Technology for choosing the subject of my thesis and for his engagement during my stay in Finland. He introduced me to the subject and guided me throughout the whole project.

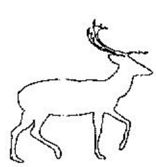
In Germany my thesis was supervised by Professor Dr. rer. nat. Holger Ihrig to whom I want to express my thanks.

All the people of the Acoustics laboratory deserve many thanks for the pleasant working atmosphere. Everybody was helpful and tried to find solutions for any problems I had. I could even manage the Finnish commands in some computer programs with the help of my colleagues.

Sampo Kolkki and the Ikaalinen Handicraft Art Institute merit thanks for providing of the pick-ups for the measurements and studies.

Finally, I want to express my appreciation to Stacey, Toomas and Jeff, who proof-read my thesis for spelling and grammar mistakes, and all of my other friends who embellished my stay in Finland.

My stay in Finland was an unforgettable experience and it broadened my mind. Besides improving my knowledge of English, it offered me the possibility to experience life in a foreign country. I was overwhelmed by Finnish nature and I liked the Finnish lifestyle. Furthermore I had the chance to meet many new friends from all over the world. Lasting friendships and all good experiences will make me visit this country again and maybe my future is in Finland.



Abstract

Since the invention of the electric guitar a large number of different pick-ups have appeared on the guitar market. Musicians sometimes do not have the essential background knowledge to understand the differences between pick-ups. For this reason a more scientific investigation is necessary. Especially the electric behaviour of the pick-up and other important details which influence the sound are worthy of study.

The first chapter of this work gives a short introduction to the subject of this diploma thesis and explains the background ideas of the task.

In the second chapter all details are mentioned which influence the sound of a guitar. It starts with a comparison between the acoustic guitar and the electric guitar. The next part deals with different types of electric guitars and with sound generation. One section treats strings and their vibration. It shows theoretically the spectra which are generated due to different kinds of excitation. Another part shows the possibilities to pick up the sound with different techniques. These techniques also include the pick-up. At the end of this chapter the basic types of pick-ups are introduced and the principle of how they work is explained.

Relevant to this thesis are the electric properties of the pick-ups and their frequency domain behaviour. Therefore, the third chapter describes the pick-up with an equivalent circuit diagram. This chapter includes theoretical calculations of different pick-ups and as well different measurements and their analysis. The measurements were done with 15 Stratocaster pick-ups and 10 different pick-ups of other types. The differences of the pick-ups are obvious and the results confirm the applicability of calculations based on the equivalent circuit diagram. At the end of this chapter investigations regarding the magnetic field of pick-ups are presented. The generation of nonlinear distortions due to the nonlinear magnetic field is shown.

Finally, the conclusion and some proposals for further research in this topic are presented in the last chapter.

Zusammenfassung

Seit der Erfindung der elektrischen Gitarre ist eine große Anzahl von verschiedenen magnetischen Gitarren-Tonabnehmern auf dem Gitarrenmarkt erschienen. Einige Musiker haben nicht das nötige Hintergrundwissen, um die einzelnen Unterschiede der Tonabnehmer zu verstehen. Dafür ist eine mehr wissenschaftliche Betrachtungsweise nötig. Besonders das elektrische Verhalten des Tonabnehmers und andere wichtige Eigenschaften, welche den Klang beeinflussen, sollten daher untersucht werden.

Das erste Kapitel ist eine kurze Einführung in das Thema der Diplomarbeit und erklärt die Gründe, die zu dieser Aufgabenstellung geführt haben.

Im zweiten Kapitel werden alle Einzelheiten erwähnt, die den Klang einer Gitarre beeinflussen. Am Anfang werden die Unterschiede zwischen der akustischen und der elektrischen Gitarre herausgestellt. Desweiteren werden verschiedene elektrische Gitarren vorgestellt und es wird die Klangerzeugung behandelt. Ein weiterer Abschnitt beschäftigt sich mit den Saiten und der Saitenschwingung. Dabei wird theoretisch die Erzeugung unterschiedlicher Spektren, aufgrund unterschiedlicher Erregung der Saiten, aufgezeigt. Der nächste Abschnitt behandelt die verschiedenen Techniken der Tonabnahme. Am Ende von diesem Kapitel werden die Grundtypen von magnetischen Tonabnehmern vorgestellt und ihre Funktionsweise wird erklärt.

Relevant sind für diese Arbeit die elektrischen Eigenschaften des magnetischen Tonabnehmers, sowie ihr Frequenzverhalten. Im dritten Kapitel wird deshalb das elektrische Ersatzschaltbild des Tonabnehmers vorgestellt. Dieser Abschnitt beinhaltet daher verschiedene Messungen mit den Tonabnehmern, sowie die Auswertung der Messungen und einige theoretische Berechnungen mit dem Ersatzschaltbild. Die Messungen wurden mit 15 Stratocaster Tonabnehmern und 10 völlig verschiedenen Tonabnehmern durchgeführt. Die Ergebnisse zeigen die Unterschiede zwischen den einzelnen Tonabnehmern und die Meßergebnisse bestätigen die Gültigkeit des Ersatzschaltbildes. Am Ende dieses Kapitels werden Untersuchungen über das magnetische Feld des Tonabnehmers durchgeführt. Dabei ist von besonderem Interesse die Erzeugung nichtlinearer Verzerrungen aufgrund des nichtlinearen magnetischen Feldes.

Zum Abschluß werden im letzten Kapitel die Schlußfolgerung und einige Vorschläge für weitere Forschungen auf diesem Gebiet präsentiert.

Lyhennelmä

Sähkökitaran keksimisen jälkeen on markkinoille tullut suuri määrä erilaisia magneettisia pick-up-tyyppisiä mikrofoneja. Kaikilla muusikoilla ja tekniikan edustajillakaan ei kuitenkaan ole riittävää perustietoa ymmärtääkseen mikrofonien eroja. Siksi tieteellinen tarkastelu on välttämätöntä. Erityisesti mikrofonien sähköinen käyttäytyminen ja muut tärkeät ääneen vaikuttavat ominaisuudet edellyttävät tarkempaa tutkimista.

Ensimmäinen luku on lyhyt johdatus diplomityöhön, jossa pyritään myös selventämään aiheen taustat.

Toisessa luvussa perehdytään kaikkiin kitaran ääneen vaikuttaviin tekijöihin. Alussa selvennetään akustisen kitaran ja sähkökitaran välistä eroa. Esille tuodaan myös erilaisia sähkökitaroita ja perehdytään äänenmuodostumiseen. Seuraavassa osassa tutkitaan kitaran kieliä ja niiden värähtelyä. Siinä osoitetaan teoreettisesti spektrin ominaisuudet riippuen kielen erilaisista herätteistä. Lisäksi käsitellään erilaisia äänen poimintatapoja. Luvun lopussa esitellään yleisimmät sähkömagneettiset pick-up-muuntimet sekä perehdytetään niiden toimintaan.

Oleellista tälle työlle ovat pick-up muuntimien sähköiset ominaisuudet ja niiden taajuuskäyttäytyminen. Siksi kolmannessa luvussa kuvataan mikrofoni ekvivalenttipiirinä. Tämä osuus sisältää erilaisia mikrofonien mittauksia, tulosten arviointia ja muutamia teoreettisia laskelmia ekvivalenttipiirin avulla. Mittaukset tehtiin 15 Stratocaster-mikrofonilla ja 10 muulla mikrofonilla. Tuloksissa erot ovat selviä ja ne tukevat ekvivalenttipiirin käyttökelpoisuutta. Luvun lopussa tutkitaan mikrofonin magneettikenttää. Kiinnostavaa on erityisesti epälineaarisen särön ilmeneminen epälineaarisesta magneettikentästä johtuen.

Viimeisessä luvussa esitetään johtopäätökset sekä muutamia ehdotuksia jatkotutkimukselle tällä alueella.

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1. Introduction

Modern light music can not be imagined without electric and bass guitars. The technical level of these instruments reached a very high standard and on the guitar market the offer is currently immense. Mainly the pick-up, the body, and the strings are responsible for the timbre of the guitar sound. Among musicians there are many discussions about the pros and cons of different pick-up models and it is necessary to know the theoretical and technical background.

For an understanding of the whole electric guitar it is not sufficient to study only the pick-up. It is also necessary to study the main differences between acoustic and electric guitars and to compare the different types of construction for electric guitars.

The sound generation of electric guitars depends not only on the electric behaviour of the pick-up but is also influenced by the strings. Mainly it is influenced by the string material and the place of excitation. A requirement to understand the string vibration is a thorough knowledge about mathematical correlations of travelling waves. The spectrum of the string vibration can be strongly influenced by musicians by the choice of string material and different kinds of excitation.

The pick-up has a major influence on the timbre of the electric guitar. Due to this it is necessary to understand the electric principles and to have a sufficient knowledge about electric circuits and magnetic fields. For the investigations it is important to describe the pick-up with an equivalent circuit diagram. It provides all requirements for the description of the electric behaviour of pick-ups. Furthermore, it is possible to calculate the influence of connected cables and amplifiers which strongly influence the electric behaviour and therefore the timbre as well. With different component values for the equivalent circuit diagram it is possible to find the differences between pick-ups. It also shows the way to manipulate the sound of pick-ups with different possibilities as a parallel connected capacitance. Another point is to distinguish single coil pick-ups from so-called humbucker pick-ups and to determine the differences.

Another aspect is to see how the magnetic field affects the electric behaviour and the sound of pick-ups. Therefore, it is necessary to know the shape of the magnetic field and to understand the basic principles of induction. A very important factor is the strength of the magnetic field density and in which manner it influences the output signal of the pick-up. It has to be distinguished between pick-ups with adjustable pole-pieces, non-adjustable pole-pieces, and bar magnets.

The target of this thesis is to provide clarity about the theoretical and technical background of pick-ups and it includes the subjects which are mentioned above.

2. Theoretical background

2.1 History of the guitar

The guitar has its origins in antiquity, probably more than 3000 years ago in Egypt. Its ancestors include the lute and the vihuela. Although the guitar appeared in several countries of Europe, it was in Spain that it developed great popularity. By the end of the thirteenth century, guitar making was a developing art in Andalusia in southern Spain. It is in the twentieth century that the guitar has become one of the most popular of all instruments, especially after the big success of the BEATLES at the beginning of the sixties of this century. In the United States alone, there are an estimated 15 to 20 million guitar players. Guitar music has developed at least five different lines: classical, flamenco, folk, jazz and rock.

Throughout the years, the guitar has undergone many changes in design, including changes in the size and shape of the body and the number of strings. Antonio de Torres Jurado developed the larger body, fan-braced sound board, and 65-cm string length that are popular today. The modern guitar has six strings tuned to E = 82.41 Hz, A = 110 Hz; d = 146.83 Hz, g = 196 Hz, h = 246.94 Hz, and e' = 329.63 Hz. The strings, which lie in a single plane, are fastened directly to the bridge. The long fingerboard is fitted with frets, which greatly simplify the playing of chords.

The top plate is usually cut from spruce, planed to a thickness of about 2.5 mm. The back is usually of a hardwood, such as rosewood, mahogany, or maple, also about 2.5 mm thick. Both top and back are braced. The bracing of the top is one of the most critical parameters in guitar design. Braces strengthen the fragile plate and also transmit vibrations of the bridge to various parts of the sound board are critical.[1]

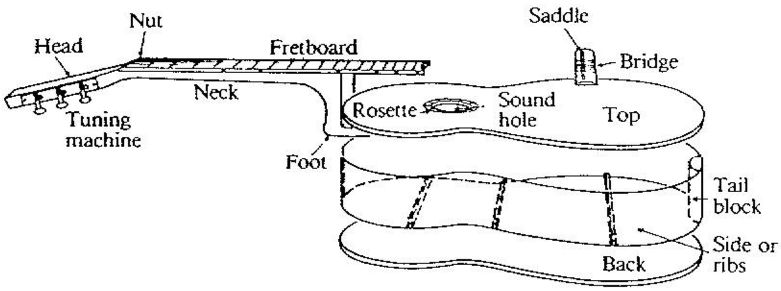


Fig. 2.1 [1] An exploded view of an acoustic guitar, showing details of construction.

[1] [6] page 201, 202 and [1] page 18, 39

2.2 The acoustic guitar

The principle by which all acoustic guitars produce musical sounds is generally agreed to be the same. When one plucks a guitar string, one applies energy to it and makes it vibrate. The string vibration alone is not sufficient to create sound waves in the surrounding air that can be clearly heard. For this reason acoustic guitars have a hollow body. The body is a carefully designed "soundbox". The guitar can be considered to be a system of coupled oscillators. The plucked string radiates only a small amount of energy directly but it excites the bridge and top plate, which in turn transmits vibrational energy to the air cavity and back plate. Sound is radiated efficiently by the vibrating plates and through the sound hole (see Fig. 2.1).

The energy of the vibrating strings is transferred to the soundbox via the saddle and the bridge over which the strings pass. The body then vibrates in sympathy with the guitar strings to create "amplified" sound waves that can be heard up to a reasonable distance from the guitar. It is the soundbox that is responsible for the guitar's projection and volume.

Fig. 2.2 is a simple schema of a guitar. At low frequencies the top plate transmits energy to the back plate and the sound hole via the air cavity; the bridge essentially acts as a part of the top plate. At high frequency most of the sound is radiated by the top plate, and the mechanical properties of the bridge become significant.

The waveform of the actual bridge force is strongly influenced by the stiffness and damping of the string and the manner in which it is plucked. If the string is plucked with a finger or a soft plectrum, the spectrum of the signal will have less prominent high harmonics.[2]

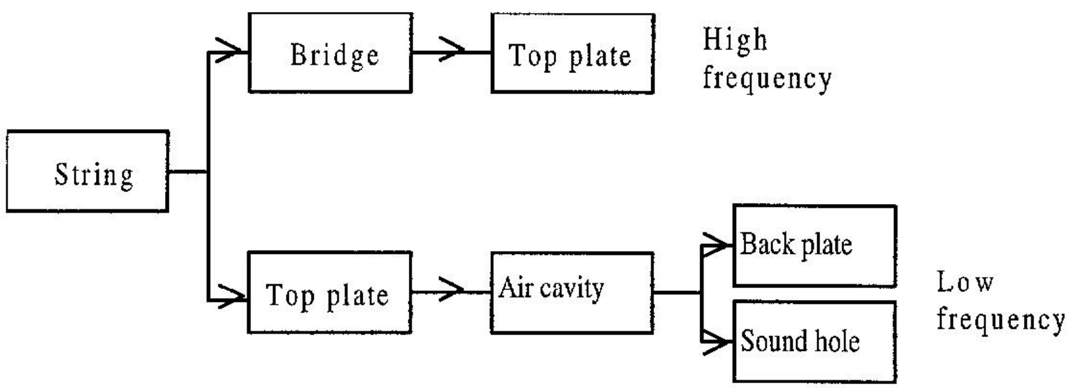


Fig. 2.2[1] Simple schematic diagram of a guitar.

[1] [6] page 203, 204
[2] [3] page 34

2.3 The electric guitar

In the original meaning, fully electric guitars of modern construction have no soundbox. Electric guitars may have a solid wooden body or a hollow body and they are made of alder, oak, ash, maple, mahogany or a combination of these woods. The solid design is the most common. In the body some blanks are left for the pick-ups, tone and volume control. The first electric guitars were made in the nineteen thirties in the USA and at that time these guitars were mainly used in jazz orchestras. The technical improvements are inseparably linked with the names Leo Fender and Les Paul. Leo Fender put his first solid body guitar on the market in 1948 and he named it "BROADCASTER" and later "TELECASTER". The "STRATOCASTER" followed in 1954 with three pick-ups instead of two (Fig. 2.3). This Stratocaster became the most played Rock'n-Roll guitar and their popularity still exists today.

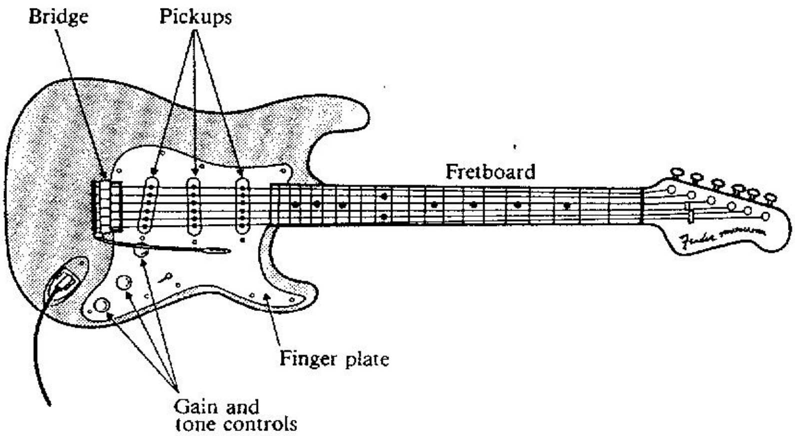


Fig. 2.3 [1] An electric guitar with three pick-ups (Stratocaster).

The first Les Paul solid guitar was presented in 1954 and he constructed this guitar in cooperation with the Gibson company. On the guitar-market is a large variety of Les Paul type guitars and we can find a lot of structural variants. In addition to these most times we can reduce different types to Stratocaster and Les Paul models. One can find several attempts at unconventional designs but they cannot supersede the traditional design. Copies of Les Paul classic guitars can be found in almost every production program of today's most important guitar makers. [2]

2.3.1 Solid body guitars

There are far more possibilities for solid guitar design than there are for acoustic guitar design. An acoustic guitar has to be constructed within certain design parameters in order to produce sufficient volume and an acceptable tone.

[1] [6] page 210
[2] [1] page 45

The shape of solid body guitars is limited only by practicality and the designer's imagination, as long as the solid body of an electrical guitar keeps the pick-ups fairly stable and provides a mounting for the necessary components.

The material used in the construction of a solid body can affect the sound of the guitar. The denser the material, the longer the natural sustain the instrument will have (not feedback assisted). The tone can be altered by changing the wood used for both the body and the neck.

Vibrations of the body have much less influence on tone in an electric guitar than in an acoustic guitar. The solid guitar, being heavier, is less susceptible to acoustic feedback (from the loudspeaker to the guitar) and it also allows the strings to continue vibrating for a slightly longer time. [1]

2.3.2 Hollow body electric guitars

There are two different kinds of hollow body electric guitars. One is the fully acoustic electric guitar and the other is the semiacoustic electric guitar. The former has two f-shaped soundholes and the height of the rib is approximately 7 to 10 cm. The top and the back are vaulted. One or two pick-ups are firmly installed here. It is also possible to play this guitar without an amplifier, but that is not common. The Gibson-models are very famous. The semiacoustic electric guitar has the same construction as the other, only the height of the rib is less (approximately 3-5cm). The handiness of this guitar is an advantage, however the sound is worse in comparison to the fully acoustic electric guitar. The play without amplifier is only for rehearsal and it is unsuitable for performance (Fig. 2.4 a). [2]

2.3.3 Semi solid electric guitars

Feedback has always been a problem with hollow body guitars. In the late nineteen fifties, Gibson brought out a range of guitars which were designed to minimise feedback. They were called thin-bodied or "semi solid" guitars to distinguish them from the ordinary deep-bodied "electric acoustic" guitars. They all had double cutaways and thin bodies with arched soundboards and backs made from laminated maple. They were true "acoustic" guitars. They were hollow and had f-shaped soundholes, therefore each had a solid block of wood set down the centre of the body. The intention was to increase sustain and prevent unwanted soundboard vibration from causing feedback. The aim of the guitars was to blend the sustain qualities of a solid-body electric with the warmer, more mellow sound produced by an acoustic instrument (Fig. 2.4 a). [3]

[1] [6] page 210
[2] [11] page 21
[3] [3] page 54

2.3.4 Electric bass guitars [1]

Leo Fender has also invented the electric bass guitar. "Precision bass" was the name of the first bass guitar (Fig. 2.4 b), and the name comes from the differences between the precision bass and the double bass. The new bass guitar was constructed with frets like a usual electric guitar and this gave the possibility for a more precise hold down of the strings. The bass guitar had a solid body as well as a correspondingly longer neck and longer string length (86 cm) and only four strings. The strings are tuned to $E_1 = 41.2$ Hz, $A_1 = 55$ Hz, $D = 73.4$ Hz and $G = 98$ Hz. Guitarists mastered the new electric bass guitar as well as double bass players. Because of this the double bass was very quickly replaced in light music with the bass guitar.

Nowadays there is again a trend to more fretless instruments. The newer invention of Ned Steinberger, the "Steinberger bass" is worth mentioning (Fig. 2.4 c). Ned Steinberger developed an instrument without the usual head and the strings are fixed with a special device. The Steinberger bass avoids top-heaviness, a disadvantage of the old bass guitars. These guitars are built with synthetic resin intensified by carbon fibre and therefore they are quite robust. The principle type of headless construction is also applicable to melody guitars. Today there are even some traditional guitars built in the headless variant.

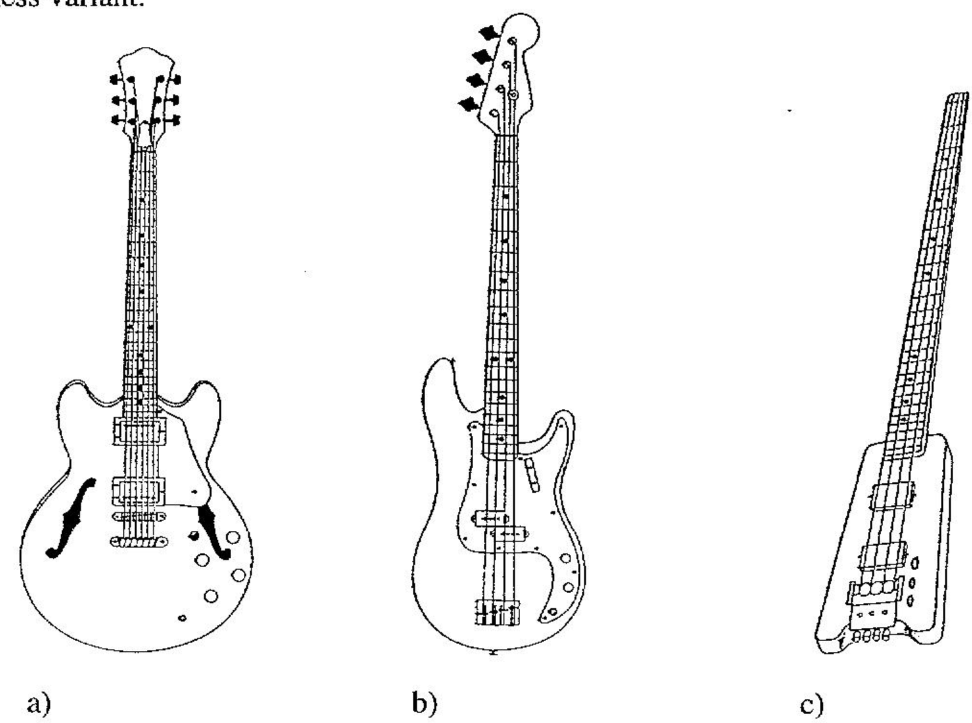


Fig. 2.4 [1] a) Hollow body electric guitar (semi solid guitars have the same appearance).
b) electric bass guitar (precision bass).
c) Steinberger bass.

[1] [1] page 40, 41, 46

2.3.5 Broadcaster, Telecaster, Stratocaster and Les Paul guitars [1]

Leo Fender tried to make a regular guitar with a clear sound but without the feedback problems associated with a vibrating soundboard. The result of his investigations was the "Broadcaster". The Broadcaster had a detachable neck similar to the banjos of this time. This was simply a question of convenience. Leo Fender felt that the neck was the part of the guitar most likely to cause problems, and his modular design meant that it could be replaced in the space of just a few minutes. The headstock was designed with all six tuning heads on one side. This made tuning easier and avoided having to fan out the strings. The Broadcaster was fitted with two single-coil pick-ups. These were wired through a three position selector switch which could be set to either the bridge pick-up, the neck pick-up, or the neck pick-up plus a special capacitor that gave a very bassy sound.

The original guitar featured the same adjustable bridge design still fitted to Telecasters today: three bolts adjust the height and the scale length of the strings in pairs. The broadcaster also had a clip-on cover-plate that snapped into place over the bridge and bridge pick-up.

Soon after the introduction of the Broadcaster its name was changed to the Telecaster. The differences between the first Broadcaster and modern Telecasters are:

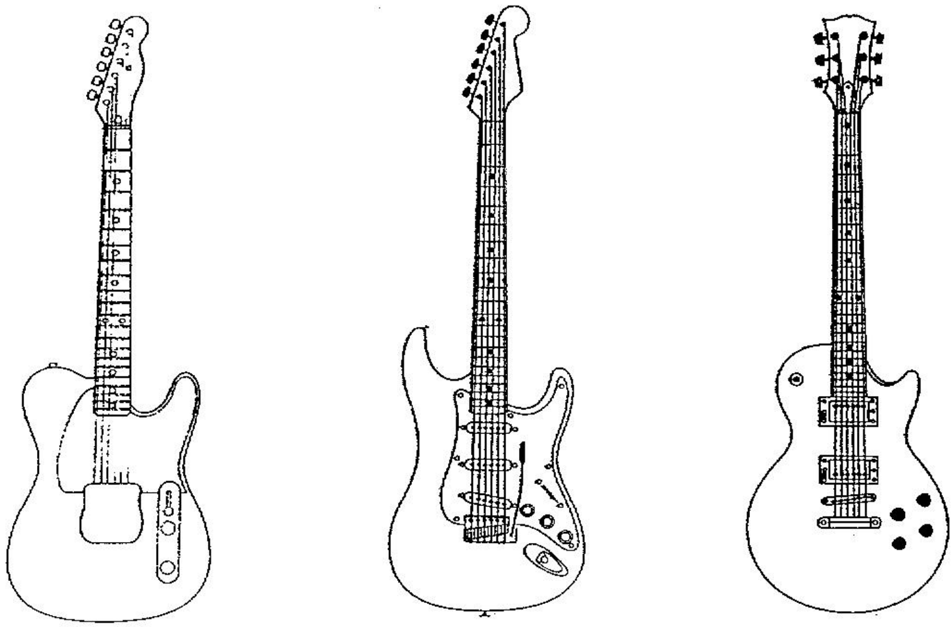
The neck truss rod is now fitted, the original black celluloid fingerplate has been replaced by laminated plastic, and the tuning heads now have slotted string posts. A number of variations on the basic Telecaster have been introduced during the last forty years (Fig. 2.5 a).

Three features immediately distinguished the Fender Stratocaster as being a revolutionary new guitar when it was introduced in 1954. First, it had a contoured, double-cutaway body designed to make it more comfortable to play. The corners were bevelled and the back had a dished recess. Second, it featured the brilliantly engineered Fender vibrato or tremolo unit built into the special floating bridge design. Third, it was the first solid body electric guitar to be fitted with three pick-ups. These were all single coils and they were wired to a three-way switch, which can be selected one at a time. However, as guitarists soon discovered, the switch could be balanced between two positions to give a unique sound. Realising the attraction of this, Fender changed the three-way switch to a five-way switch. Like the Telecaster, the Stratocaster has remained virtually unchanged and it is available in various different finishes and colours. (Fig. 2.5 b)

The story of Gibson guitars goes back into the nineteenth century. Gibson pioneered the arch-top acoustic guitars and also the hollow body electric guitar. Their involvement with solid-body guitars came slightly later (after Fender's introduction of the Broadcaster) and owes a lot to the renowned Les Paul. Les Paul wanted the guitars to have a natural twenty-second sustain and this is the reason why they are relatively heavy. The body was made from solid mahogany with a 12 mm maple cap or facing on top. The profile of the carved top was a suggestion of Gibson intended to make it hard to copy.

[1] [3] page 56, 57

The original 1952 Gibson Les Paul had two high-impedance single-coil pick-ups with cream cover plates. It was fitted with a three-way pick-up selector switch and separate volume and tone control. At first Les Paul used his own distinctive trapeze tailpiece and bridge. This caused problems, and in 1955 it was replaced by the new, adjustable bridge and tailpiece. The height of the whole bridge can be raised or lowered, but each string sits on its own individually adjustable saddle. In 1957, the single coil pick-ups were replaced by the new humbucking pick-ups.



a) b) c)

Fig. 2.5 [1] **a)** Telecaster with single-coil bridge pick-up.
b) Stratocaster with three single-coil pick-ups.
c) Les Paul type with two humbucking pick-ups.

2.3.6 The Principle of sound generation in guitars [1]

Fig. 2.6 illustrates the transmission of the sound from the generation (with consideration of different possibilities to pick up the sound) to the listener. In principle, there is no distinction between the acoustic and electric guitar in the sound generation, due to their constructive features. Due to the different sound pick up as well as differently processing of the sound, they have differences in sound.

The real sound generation occurs when the strings are plucked, because the strings get energy from the finger. This energy excites the strings to vibrations. The vibration shape characterises the sound pattern and it depends on the properties of the strings (diameter, material, structure etc.).

[1] [1] page 46-48

There are a lot of different possibilities to excite the strings (pluck, strike, play with or without plectrum etc.). The sound is also affected by the manner of excitation.

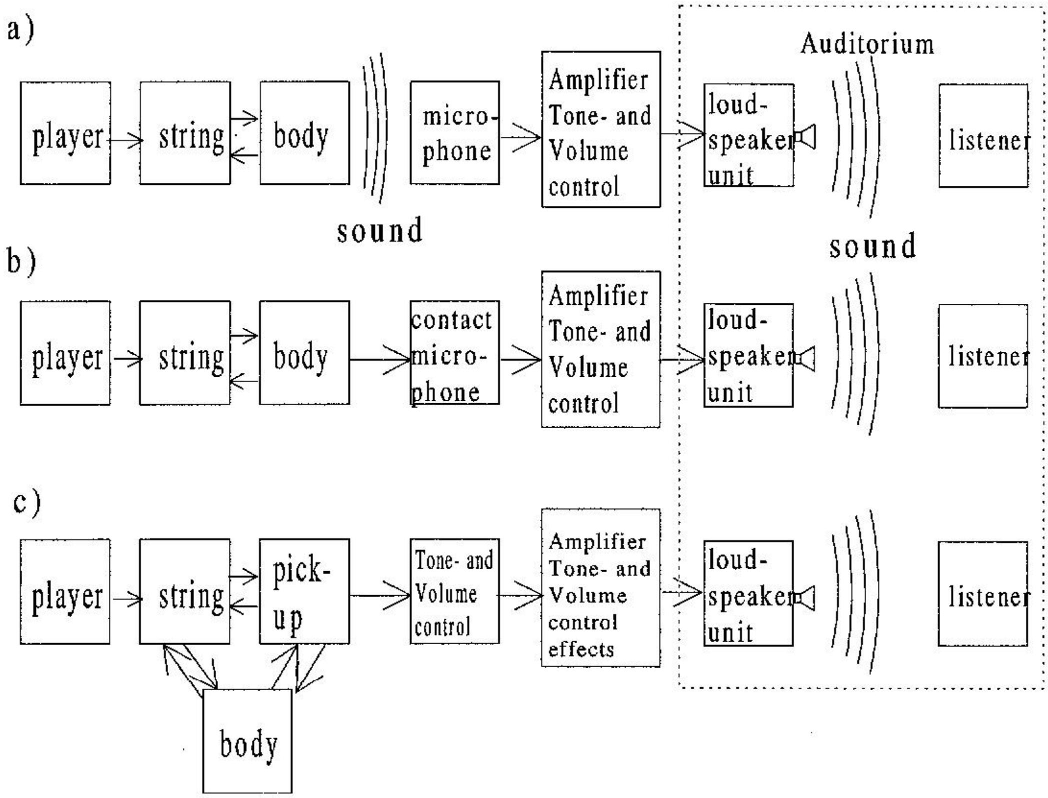


Fig. 2.6 [1] Simple representation of sound transmission from electric guitars and electrified acoustic guitars to the listener.
Pick up of the sound by:
a) Microphone.
b) Contact microphone on the top of the guitar.
c) Pick-ups (directly from the strings).

[1] [1] page 46

The strings transfer the vibration to the body through the bridge. Between the strings and the different parts of the guitar, such as the body, bridge, saddle and neck, exist an interaction. This system can be considered as a coupled vibration-system. These effects are desired when one plays an acoustic guitar because the strings radiate only a small amount of the sound. Only the coupling, with an extensive resonant cavity, is enabling a considerable sound radiation. The small surface of the strings is the source of this problem, because the strings cannot excite the surrounding air to vibration. At very low frequencies an acoustic short circuit exists. The air flows only around the string during an acoustic short circuit and the raising of pressure on one side of the string extinguishes the diminishing of pressure on the other side.

The properties of the resonant body, the cavity of the guitar and its construction are responsible for the acoustic pattern. Electric guitars with solid bodies do not need a resonant body in the original meaning. They directly convert the mechanical vibration into electric vibration and because of this the sound radiation is not interesting. It is not possible to avoid completely the vibration of the body. Therefore, the timbre and the decay characteristics depend on the used timber, the shape of the bridge and several other design features. The semiacoustic electric guitar has an intermediate position between the fully acoustic electric guitar and the solid body guitar. There is a small coupling between strings and resonant body. The amount of radiated sound from semiacoustic electric guitars is less than the radiated sound from acoustic guitars.

The idea of the "electrification" of the guitar was to magnify the volume and to change the tone with different possibilities. We can simplify the electrification to three basic principles.

1) The radiated sound of the guitar is recorded by a suitable microphone.

2) The vibration is sensored on the top plate by a contact microphone (piezoelectric microphone) and converted into electrical vibration.

3) The vibration of the string is directly converted into electric vibrations with the help of a pick up (The most common application).

After the conversion we have an electric signal according to the mechanical vibrations of the strings. The signal is an alternating voltage with all frequencies of the string vibration. The timbre is changed by a tone control circuit and the resultant signal supplies a power amplifier and then the loudspeaker. Not all pick-ups and microphones are able to generate a sufficient output-voltage. These devices need an preamplifier because of the transmission loss. Sometimes this preamplifier is combined with an active tone control unit.

The last part of the transmission of sound to the listener is the room acoustics. The same guitar with the same amplifier sounds quite different in two acoustically unequal rooms. There are often no possibilities to influence the room acoustics. However, it is possible to influence pick-up, microphone, amplifier, tone- and volume control. Generally the musicians have to live with the properties of the room.

The acoustic pattern can be altered quite strongly by the transmission properties of all devices. Every part of the sound transmission is important for the sound. The quality of every device decides if it is possible to reach a desirable sound. More quality in one device cannot compensate the loss of quality in another device. There are several possibilities to generate far more different sounds with special effect-circuits such as phasing, flanging, distortion, hall, echo and more. Sometimes these effect-circuits are directly installed into the amplifier or even into the guitar. They are available as a special device, at least.

It is also necessary to mention the acoustic feedback. It occurs when a microphone or transducer (such as the pick up of an electric guitar) picks up its own amplified sound and feeds it through the sound system once again. Often this is a problem but sometimes it is utilised as a special effect.

2.4 The strings [1]

In the past, guitar strings were made of either wire or gut (called "cat gut" but, in fact, almost always the intestines of sheep). However, modern guitar strings can be divided into two basic types: steel and nylon. Steel strings are used on electric guitars and on flat-top and arch-top acoustic guitars; nylon string are used on classical and flamenco guitars.

Most guitars are strung with a set of six strings, each of a different thickness and each tuned on a different pitch. Of these six, the first and second strings are "plain", and the forth, fifth and sixth strings are "wound". On electric guitars, the third string may be either plain or wound.

It is impractical to make thick plain strings. The mass of the bass strings is increased by wrapping lengths of extra wire around a central core. These are wound strings. The central core may be either round or hexagonal. With steel strings, the core is made of steel; with nylon strings the core is made of nylon.

There are various materials to produce the wire winding. They can be roughly classified as "white" or "silver" metal (stainless steel, chrome, nickel, nickel alloy, silver-plated copper) or "gold" or "yellow" metal (bronze, brass and various alloys). Either white or yellow strings can be used on acoustic guitars, although most players prefer the yellow bronze or brass strings. Only certain strings can be used on electric guitars with magnetic pick-ups. These are the magnetically responsive white metal ones. It is preferred to use highly permeable ferromagnetic materials like nickel or certain stainless chrome alloys.

[1] [3] page 162

All steel core strings are suitable for electromagnetic transducers. Stronger strings generate a louder sound because the vibrating mass is heavier. Neither yellow metal strings nor nylon strings will work with a magnetic pick-up. A guitar with a contact transducer or microphone can be fitted with any type of string.

The shape or profile which the winding gives the wound string varies according to whether the string is roundwounded, flatwounded or groundwounded.

2.4.1 Roundwound strings

Roundwounded strings are the most commonly used strings. To produce the bottom three (or four) wound strings, the steel or nylon core is wrapped with a long, continuous length of round wire. The winding is done automatically. Roundwound strings produce a good tone and volume, and when new they give a clear sound suitable for both acoustic and electric guitars.

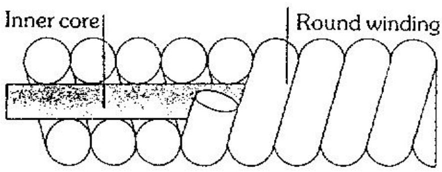


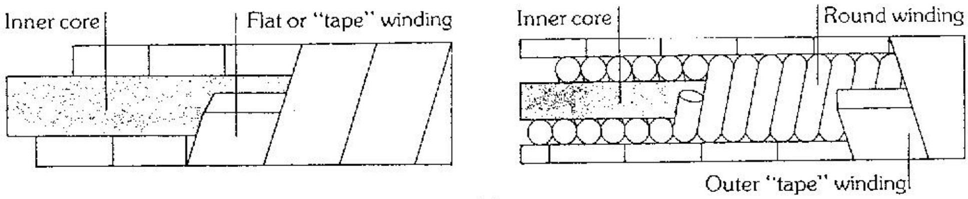
Fig. 2.7 Roundwound string.

2.4.2 Flatwound strings

The flatwound string, also known as "tapewound" strings, have a far smoother surface than that of roundwound strings. This is because the winding is made, not from round wire, but from flat metal tape or ribbon.

Flatwound strings were designed to overcome the problem of "finger squeak". This noise occurs when the guitarist's left hand moves up and down the fingerboard while in contact with the strings. The smooth, flat surface of flatwound strings helps to reduce this noise. Flatwound strings have a more mellow sound than roundwound strings and therefore they are often preferred by jazz players.

Some flatwound strings are made with both a round and a flat winding. The round winding goes on first and is then covered with a flat ribbon winding, usually known as "compound flatwounds".



a) b)

Fig. 2.8 a) Flatwound string.
b) combination of flatwound and roundwound string.

2.4.3 Groundwound strings

These strings are an attempt to combine the different advantages of roundwound and flatwound strings. They are made in the same way as roundwound strings but the winding is then ground down and polished so that the protrusions are removed and a flattened surface is left. Groundwound strings therefore give some of the bright tone quality, projection and sustain of roundwound strings while also offering the smoother feel of flatwounds.

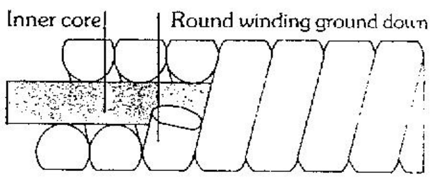


Fig. 2.9 Groundwound string.

2.4.4 The thickness of strings [1]

The structure of plain strings is only limited by the choice of suitable diameters. The wound string usually has a grading of the thickness. Most manufacturers apply windings in the strength of heavy tension, medium tension and light tension. Electric guitar strings are even available in finer grading such as "extra light", "super light" and "ultra light". Table 2.1 shows an example for guitar strings. Internationally it is usual that the diameter is indicated in inch and in mm (1 inch = 25.4 mm). In the table the tractive power of the strings (in Newton [N]) is also listed.

[1] [1] page 59

Light strings are easier to push down onto the frets and make note bending easier. They can be hard to keep in tune, they give less volume and sustain, and one may find that one tends to bend strings out of tune accidentally when playing chords. Heavy strings have all the opposite effects. The guitar may be slightly harder to play, but the strings will distort less when playing chords, will hold their pitch better, and will give longer sustain and more volume.

Electric bass strings are much thicker than the guitar strings. The winding exists for up to four layers and the outer layer is a round wire or a smooth steel or nylon band.

STRINGS									
	e' (I)			h (II)			g (III)		
indication	inch	mm	N	inch	mm	N	inch	mm	N
ultra light	0.008	0.20	46.1	0.010	0.26	43.2	0.015	0.38	57.9
super light	0.009	0.23	60.8	0.011	0.28	50	0.016	0.41	67.7
extra light	0.010	0.26	77.5	0.013	0.33	69.6	0.017	0.43	73.6
light	0.011	0.28	89.2	0.014	0.36	83.4	0.020	0.51	93.2
light medium	0.012	0.31	109	0.016	0.41	107	0.022	0.56	109.9
medium	0.013	0.33	125	0.017	0.43	117.7	0.024	0.61	127.4
medium heavy	0.014	0.36	148	0.018	0.46	124.7	0.028	0.71	171.7

STRINGS									
	d (IV)			A (V)			E (VI)		
indication	inch	mm	N	inch	mm	N	inch	mm	N
ultra light	0.022	0.56	60.8	0.030	0.76	61.8	0.040	1.02	61.8
super light	0.024	0.61	71.6	0.032	0.81	71.6	0.042	1.07	66.7
extra light	0.026	0.66	83.4	0.034	0.86	79.5	0.044	1.13	74.6
light	0.030	0.76	107.9	0.038	0.97	102	0.048	1.22	87.3
light medium	0.032	0.81	122.6	0.040	1.02	110.9	0.050	1.27	97.1
medium	0.034	0.86	138.3	0.042	1.07	120.7	0.054	1.37	106.9
medium heavy	0.036	0.92	156.9	0.044	1.13	137.3	0.056	1.42	116.7

Table 2.1 [1] Strength, diameter and tractive power of roundwound strings (example).

[1] [1] page 58

2.5 Physical basis of string vibration

The frequency produced by a guitar string depends on three things:
1) The tractive power k [N] placed on the string (this is controlled by the tunin heads).
2) The length l [m] of its vibrating section (determined by the distance between the nut and the bridge).
3) The mass m [kg] of the string (depends on material and thickness).

The equation for the fundamental tone can be written as:

Mersenne's law: $$f_n = \frac{n}{2l}\sqrt{\frac{kl}{m}} = \frac{n}{2l}\sqrt{\frac{\sigma_z}{\rho}}.$$ **Eq. (2.1-1)**

n is the number of the harmonic (natural multiple).

σ_z $[\mathrm{N/m^2}]$ is the tension on the string.

ρ $[\mathrm{kg/m^3}]$ is the density of the string material.

The vibration of a string consists of its fundamental and its partials. The partials are in the ideal case natural multiples of the fundamental and this can be called harmonics.

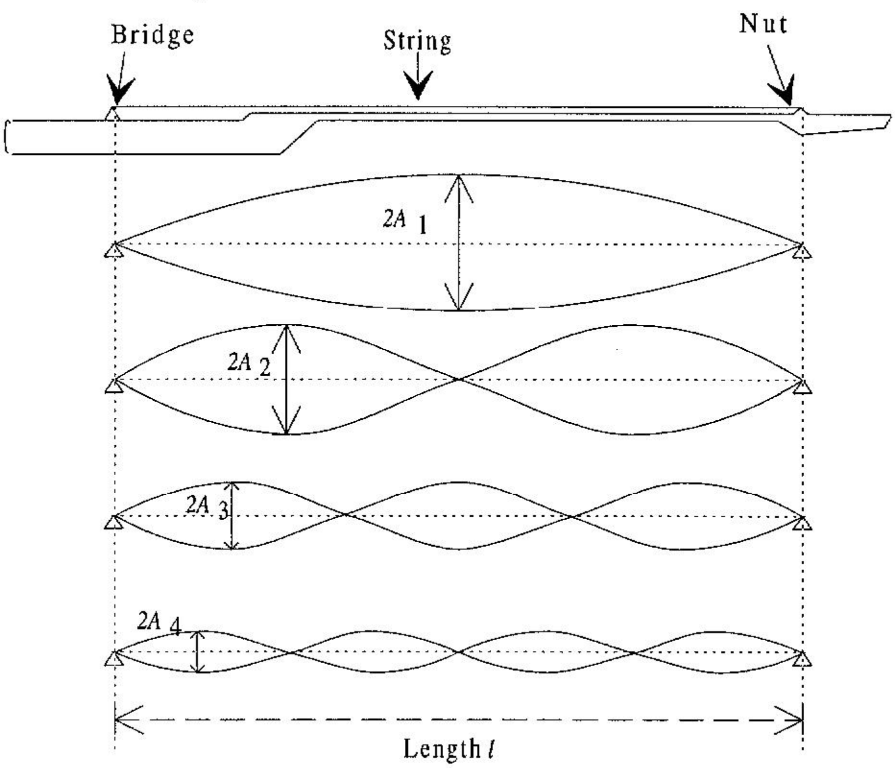


Fig. 2.10 Fundamental and three harmonics. Representation of amplitude A, wave length $\lambda = x*l$, fundamental $x = 2$, 2nd harmonic $x = 1$, 3rd harmonic $x = 2/3$, 4th harmonic $x = 1/2$.

The string forms standing waves after plucking. The string length l is an even multiple of the half wave length $\lambda/2$. The amplitudes of the fundamental and the harmonics determine the timbre of the sound and they depend on:

1) The strength of plucking.
2) The place of excitement on the string
3) The used string type.
4) The body of the guitar.
5) The time.

The amplitudes decrease exponentially with the time. Their dependency on the time is expressed by the following equation.

$$A_i(t) = A_i(t=0) \cdot e^{-\beta_i t}. \qquad \textbf{Eq. (2.1-2)}$$

The coefficient β_i is the damping rate of the different vibration components. The different coefficients are unequal, therefore, the duration of the components is also unequal. The damping of the higher harmonics is mostly stronger than the damping of the fundamental and lower harmonics. [1]

2.5.1 The bending of stiff strings [2]

One cannot ignore the stiffness of the strings when they have a big diameter as well as the string length is short. Therefore, the partials are not exactly natural multiples of the fundamental. One has to consider the stiffness with a correction factor. The equation for the stiff string can be written as: [2]

$$f_n = \frac{n}{2 \cdot l}\sqrt{\frac{\sigma_z}{\rho}\left[1 + \frac{n^2 \cdot \pi^2}{16}\left(\frac{d}{l}\right)^2 \cdot \frac{E}{\sigma_z}\right]}. \qquad \textbf{Eq. (2.1-3)}$$

d [m] is the diameter of the string.
E [N/m^2] is the elastic modulus of the string material.

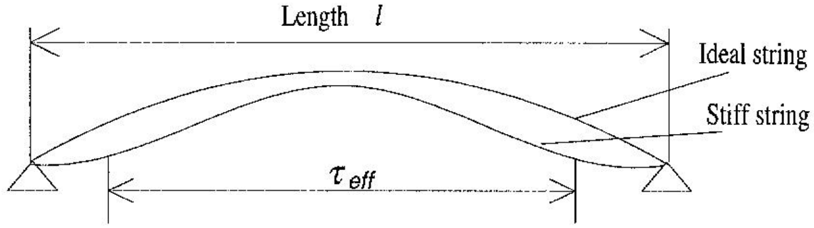


Fig. 2.11 The shorter string length of stiff strings compared with ideal strings (Extreme representation).
τ_{eff} is the effective string length of stiff strings.

[1] [11] page 17, 18
[2] [1] page 51

The series of sounds are felt impure when the partials are not exactly natural multiples of the fundamental. The ratios $f_2 = 2*f_1$, $f_3 = 3*f_1$ etc., are only correct in the physical ideal case but the experience shows other behaviour of the strings during the vibration. The reason is not only the stiffness of the strings, also a nonuniform mass distribution and a unequal dirtying on the string can cause such problems. The stiffness causes a seemingly shorter string length because the bending radius of stiff strings is slightly bigger (Fig. 2.11). This generates a higher frequency compared to the theoretical Eq. (2.1-1) (Fig. 2.12). The frequency of every partials is slightly higher than the natural multiples of the fundamental and this may sound quite unpleasant. The thicker and the shorter the strings are, the more the stiffness effects the sound. [1]

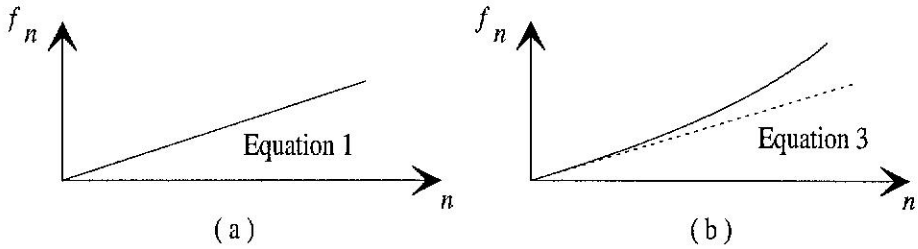


Fig. 2.12 Graphs of f_n versus n for (a) an ideal string, (b) a stiff string.

2.5.2 Calculation of the amplitudes

A string of a guitar has a finite length l and the string is fixed on both ends. It is convenient to study standing waves for this case. For instance, it is possible to describe two sinusoidal waves with equal amplitudes moving in opposite directions: [2]

$$\xi(x,t) = A\sin\left(\omega t + \frac{2\pi x}{\lambda}\right) - A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right). \qquad \textbf{Eq. (2.2-1)}$$

ξ is the displacement in position x, at a time t and λ is the wave length. With familiar trigonometric identities, it can be written as: [2]

$$\xi(x,t) = 2A\cos(\omega t)\sin\left(\frac{2\pi x}{\lambda}\right). \qquad \textbf{Eq. (2.2-2)}$$

This expression has two important properties. First, there are some values of x where ξ remains zero at all time; whenever the crest of one travelling wave component arrives there, it is always cancelled by a trough of the other. Such points are called nodes of the standing wave. Second, there are some values of t for which ξ is zero simultaneously at all locations. At that moment the velocity v has its greatest value.

[1] [11] page 19
[2] [5] page 105-108

In this case all parts of the system oscillate in unison with simple harmonic motion of the same frequency. Such a motion is called normal mode of the system. A trial solution representing this motion is: [1]

$$\xi(x,t) = \cos(\omega_n t - \varphi_m) \cdot h(x) \qquad \textbf{Eq. (2.2-3)}$$

(with: $\omega_n = 2\pi f_1 n$).

This equation has to satisfy Newton's second law (basic law of dynamic): [1]

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}. \qquad \textbf{Eq. (2.2-4)}$$

c is the speed with which waves travel along the string.

This is only possible if the unknown function *h(x)* obeys the following equation: [1]

$$-\omega^2 h(x) = \frac{c^2 d^2 h(x)}{dx^2}. \qquad \textbf{Eq. (2.2-5)}$$

This would be satisfied throughout the interior of the string by any combinations with Eq. (2.2-6): [1]

$$h(x) = C\sin\left(\frac{\omega_n x}{c}\right) + D\cos\left(\frac{\omega_n x}{c}\right). \qquad \textbf{Eq. (2.2-6)}$$

C and *D* are arbitrary constants.

Both ends of the strings are fixed (boundary conditions Fig. 2.13): [1]

$$\xi(0,t) = 0 \text{ and } \xi(l,t) = 0. \qquad \textbf{Eq. (2.2-7)}$$

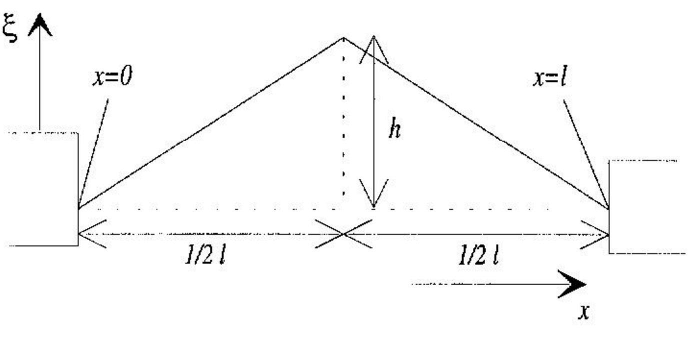


Fig. 2.13 [2] An example of boundary conditions for a stretched string.

[1] [5] page 108, 109

[2] [4] page 195

The only way to meet the first condition is to set *D=0*. Then the second condition allows the amplitude *C* to be anything as long as one requires that f_n is a natural multiple of f_1. Normal mode frequencies that form a harmonic series are a very special feature of one-dimensional systems whose properties are uniform everywhere along their length. A nonuniform mass distribution on a string will have a nonharmonic list of mode frequencies. The general motion of a string must be some linear combination of its normal modes, and that, given the boundary conditions. The typical normal mode solution is (from Eq. (2.2-3) and Eq. (2.2-6)). [1]

$$\xi(x,t)=\sum_{n=1}^{\infty} C_m \cos(\omega_n t-\varphi_m)\sin\left(\frac{n\pi x}{l}\right). \qquad \textbf{Eq. (2.2-8)}$$

With familiar trigonometric identities, the Eq. (2.2-8) can be written as. [1]

$$\xi(x,t)=\sum_{n=1}^{\infty}\sin\left(\frac{n\pi x}{l}\right)\left(C_n\cos(\omega_n t)+S_n\sin\omega_n t\right). \qquad \textbf{Eq. (2.2-9)}$$

The coefficients C_n and S_n depend on the initial conditions of the string. These are the positions and velocities of each point on the string at time *t=0*. These vary according to the particular problem. Now it is possible to take the problem of a string constrained into some shape described by the function $\xi(x,0)$, and then, at a time *t=0* released. The initial velocity of every point on the string is zero. The initial conditions can therefore be summarised by the fact that the initial shape is $\xi(x,0)$ and the velocity is

$$\left[\frac{\partial\xi(x,0)}{\partial t}\right]=0. \qquad \textbf{Eq. (2.2-10)}$$

For all values of *x* the velocity must be zero and it follows that all the coefficients S_n must be zero. Then the result of the differential Eq. (2.2-10) is [1]

$$\xi(x,t)=\sum_{n=1}^{\infty} C_n\sin\left(\frac{n\pi x}{l}\right)\cos(\omega_n t). \qquad \textbf{Eq. (2.2-11)}$$

At the time *t=0*, this equation becomes [1]

$$\xi(x,0)=\sum_{n=1}^{\infty} C_n\sin\left(\frac{n\pi x}{l}\right). \qquad \textbf{Eq. (2.2-12)}$$

[1] [4] page 199, 200

To find the coefficient C_n, one follows the methods of the Fourier-analysis. Finally, one gets the solution as an equation and it can be written as [1]

$$C_n = \frac{2}{l}\int_0^l \xi(x,0)\sin\left(\frac{n\pi x}{l}\right)dx. \qquad \textbf{Eq. (2.2-13)}$$

So in the case of zero initial velocity, all one needs to calculate the C_n of Eq. (2.2-11) is to use the knowledge of the initial shape of the string, $\xi(x,0)$, together with the integral in Eq. (2.2-13).

[1] [4] page 200, for more details, page 195-200
[5] for more details, page 38, 39, 103-107
[2] for more details, page 35, 36, 55, 61, 62

2.5.3 Vibrations of a plucked string (modes)

When the string of a guitar is excited by plucking, the resulting vibration can be considered to be a combination of the normal modes of vibration or resonances. For example, if the string is plucked at its centre, the resulting vibration will consist of the fundamental plus the odd-numbered harmonics. Fig. 2.14 illustrates how the modes associated with the odd-numbered harmonics, when each is present in the right proportion, can add up at one instant in time to give the initial shape of the string. Modes 3, 7, 11, etc., must be opposite in phase from modes 1, 5 and 9 in order to give a maximum displacement in the centre.

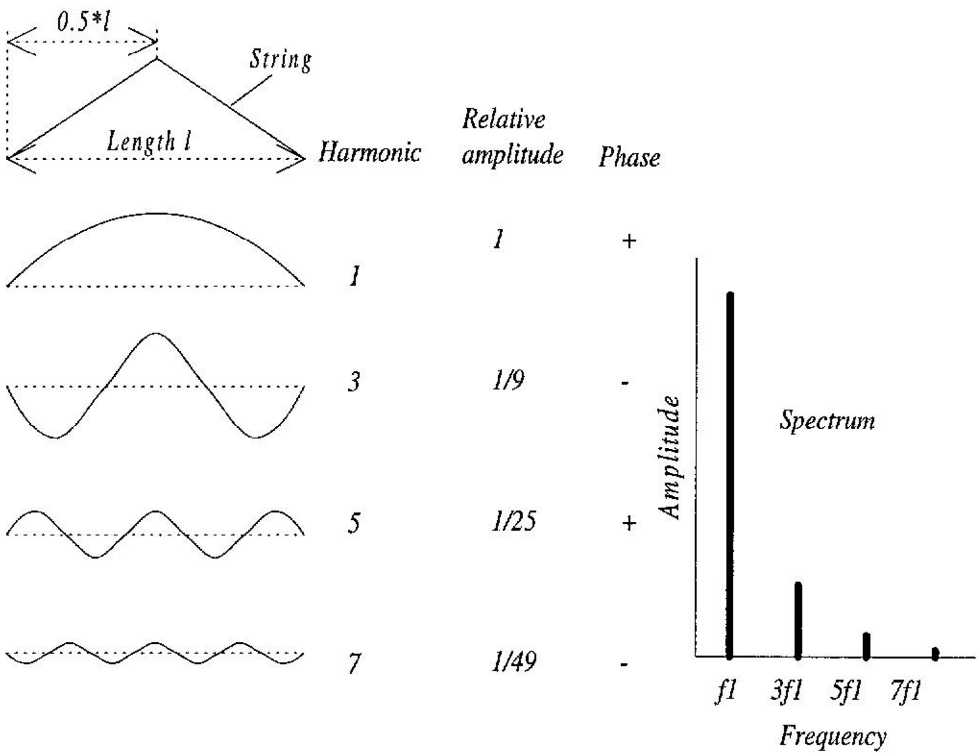


Fig. 2.14 [1] Shape of the string before release. Odd-numbered modes of vibration add up in appropriate amplitude and phase to the shape of a string plucked at its centre.

If the string is plucked at a point other than its centre, the recipe of the constituent modes is different. For example, if the string is plucked one-fifth the distance from one end, the recipe of mode amplitudes is as shown in Fig. 2.15 In this case the fifth harmonic is missing. Plucking it one-fourth the distance from the end suppresses the fourth harmonic and so on. In Fig. 2.14 it can be noted that plucking it at one-half the distance eliminated the second harmonic, as well as all other even harmonics. [2]

[1] [1] page 53
[2] [6] page 189 and [11] page 53

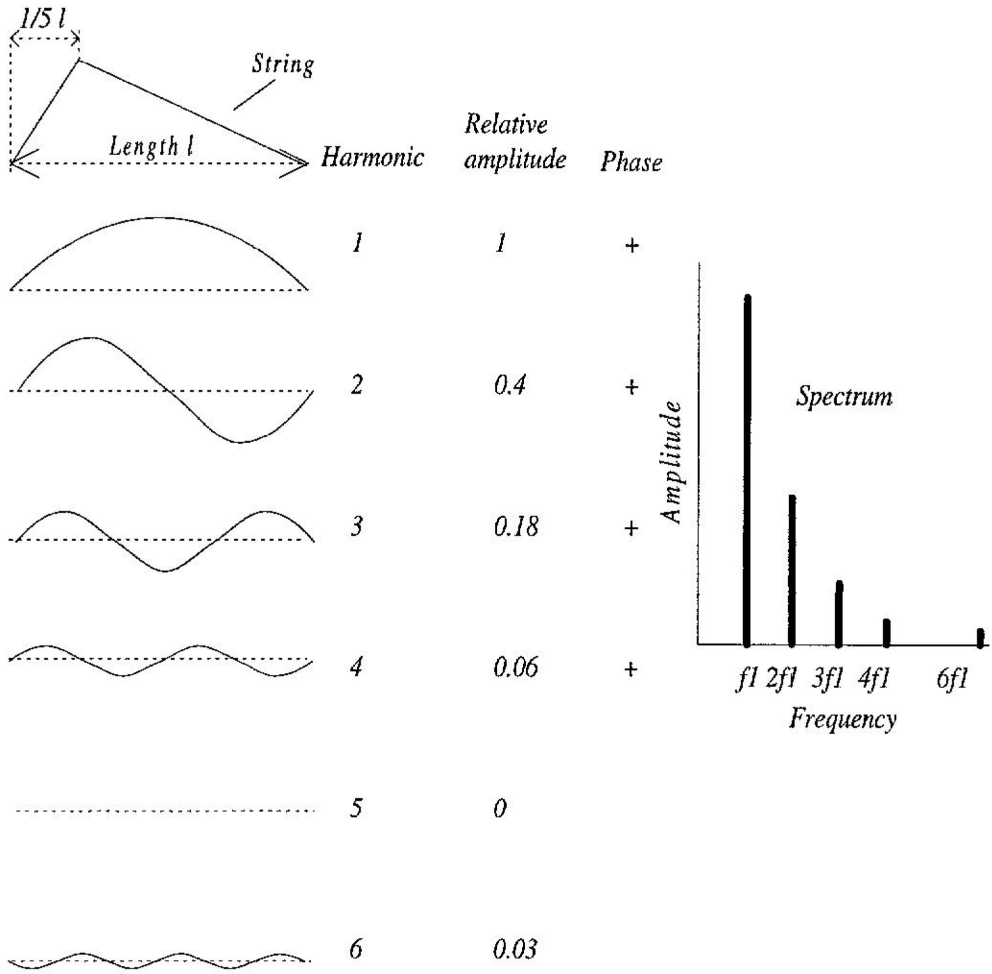


Fig. 2.15 [1] The addition of modes to obtain the shape of a string plucked at one-fifth its length. It should be noted that the spectra in Fig. 2.14 and the figure above show the relative amplitudes of the different modes of vibration. The spectra of the radiated sound will have the same frequencies but their relative amplitudes will be quite different due to the acoustical properties of the instrument.

In the practical play of a guitar, one plucked the string approximately at one-fifth the distance from one end. The 5. mode and all natural multiple of it are missing in the spectrum in this case. When one compares the spectra it is obvious that in Fig. 2.14 the fundamental comprises the most energy and the harmonics are very weak. If the excitation is closer to the bridge the harmonics are stronger and the sound has a sharper timbre. [1]

[1] [1] page 53

In general one can get the solution for the amplitudes C_n with Eq. (2.2-13). If one excites the string at a distance l_1 from one end, it is possible to calculate all amplitudes C_n with the following expression.

$$C_n = \frac{2h}{(n\pi)^2} \cdot \frac{l^2}{l_1(l-l_1)} \cdot \sin\left(\frac{n\pi l_1}{l}\right). \qquad \textbf{Eq. (2.3-1)}$$

h is the displacement at the excited point of the string (Fig. 2.13).

Since all the modes shown in Fig. 2.14 have different frequencies of vibration, they quickly get out of phase, and the shape of the string changes rapidly after plucking. The shape of the string at each moment can be obtained by adding the normal modes at that particular time, but it is more difficult to do so because each of the modes will be at a different point in its cycle. The resolution of the string motion into two pulses that propagate in opposite directions on the string, which is called time analysis, is illustrated in Fig. 2.16. It is shown a time analysis of the string plucked at 1/5 of its length. A bend racing back and forth within a parallelogram boundary can be viewed as the resultant of two pulses travelling in opposite directions. No losses of the waves are assumed to occur.

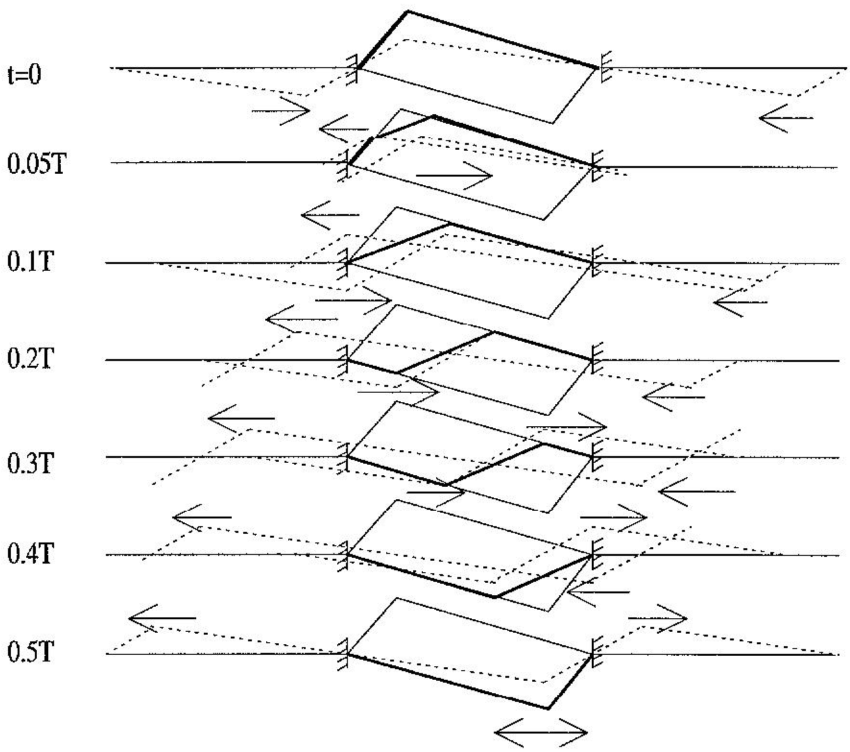


Fig. 2.16 [1] Time analysis through one half cycle of the motion of a string plucked one-fifth of the distance from one end. The motion can be thought of as due to two pulses moving in opposite directions (dashed curves). The resultant motion consists of two bends, one moving clockwise and the other counterclockwise around a parallelogram.

[1] [2] page 42

2.6 The principles of sound pick up

"Transducer" is the name given to any electronic or electro-magnetic device used to convert forms of physical energy into electrical energy. All guitar pick-ups are transducers. They convert the energy produced by the vibrating guitar strings into AC electrical pulses that are fed to an amplifier. The amplifier magnifies these pulses by many times before the loudspeaker transforms them back into sound waves. There are three different possibilities to convert the string vibration into electric pulses. These are, as seen in section 2.3.6, performed by:

1) Conventional microphones.
2) Contact microphones.
3) Magnetic pick-ups.

In the following sections it will be described how the different sound pick up devices work. [1]

2.6.1 Microphones

A microphone is a transducer that produces an electrical signal when excited by sound waves. Microphones are designed to respond to variations in air pressure due to the sound wave or to variations in particle velocity as the sound waves propagate. Most microphones in common use are pressure microphones.

A pressure microphone has a thin diaphragm, which moves back and forth with the rapid pressure changes in a sound wave. The diaphragm is connected to some type of electrical generator, which may be a piezoelectric crystal (crystal microphone), a moving coil (dynamic or magnetic microphone), or a variable capacitor (condenser microphone). [2]

2.6.1.1 Crystal microphones [2]

Crystal microphones (see Fig. 2.17) use piezoelectric crystals that generate a voltage when sound pressure deforms the diaphragm. The amount of charge that appears due to the deformation is proportional to the vibration of the diaphragm and disappears when the force is removed. If leads are attached to the crystal at the appropriate places, an electrical output signal is obtained.

Crystal microphones have a comparatively large electrical output voltage, which makes them convenient for use in portable sound equipment and tape recorders. The high-frequency response of most crystal microphones is less flat than that of condenser microphones due in part to the greater mass that must be moved. Therefore, they are not often used in the professional sound recording.

[1] [3] page 52 and [1] page 71
[2] [1] page 72 and [6] page 394

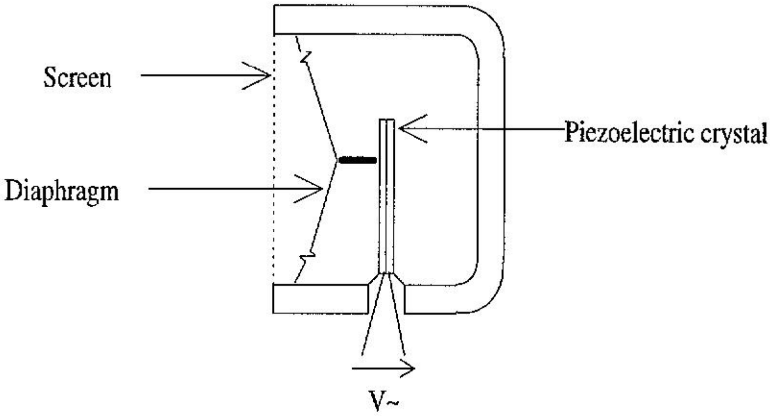


Fig. 2.17 [1] Crystal microphone.

2.6.1.2 Dynamic microphones [1]

In a dynamic or magnetic microphone (see Fig. 2.18), an electrical signal is generated by the motion of a conductor in a magnetic field. This is an example of electromagnetic induction, or the dynamo principle. In the most common type of dynamic microphone, sound pressure on a diaphragm causes the attached coil to move in the field of a magnet. Movement of the coil thus generates an output voltage.

$$u_l = \oint \vec{B} \cdot \left(\vec{v} \times d\vec{l}_c \right). \qquad \textbf{Eq. (2.4-1)}$$

u_l is the induced voltage.
B is the magnetic flux density.
v is the speed of the coil.
l_c is an element of the coil.

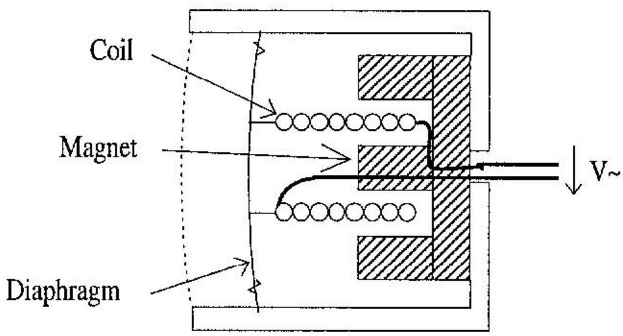


Fig. 2.18 [1] Dynamic microphone.

[1] [1] page 72, 73 and [6] page 394

The voltage generated by a dynamic microphone is quite small, but the impedance of the coil is also small. The power generated by a dynamic microphone is not necessarily weak, even though the voltage output is low. It is essential that the mass of the moving coil is kept very small in order to yield good response at high frequencies.

2.6.1.3 Condenser microphones [1]

When two metal plates, that face each other, are connected to a battery, they acquire and store electric charge. Such an arrangement of charged plates is called a capacitor, or condenser. The amount of electric charge that can be stored in a condenser depends on the size of the plates and on their spacing. The capacitance of two plates can be written as: [2]

$$C = \frac{Q_c}{U_c} = \frac{\varepsilon_0 \varepsilon_r A_P}{d_p}. \qquad \textbf{Eq. (2.4-2)}$$

Q_c is the quantity of electricity.
U_c is the voltage at the capacitance.
ε_0 is the relative permittivity.
ε_r is the absolute permittivity.
A_p is the area of the plates.
d_p is the distance between the plates.

In a condenser microphone (see Fig. 2.19), one plate is usually the thin movable diaphragm and the other is the fixed backing plate. If the diaphragm starts to vibrate the distance d_p alters according to the sound pressure. In consideration of the alternating distance the capacitance starts to change the values. The charge Q_c on the condenser can be assumed to be a constant because of the big resistance R. The DC circuit has a very high time constant $T_{con}=RC$.

The alternating voltage u_g on the condenser can be expressed as:

$$u_g = \frac{Q_c}{\varepsilon_0 \varepsilon_r A_p} \cdot \frac{d\left(\Delta d_{p(t)}\right)}{dt}. \qquad \textbf{Eq. (2.4-3)}$$

A condenser microphone has a very high source impedance, and for this reason, a preamplifier is usually incorporated into the microphone itself. The diaphragm can be made very light, therefore, the condenser microphone is capable of excellent response at high frequencies. A major disadvantage is the need for a relatively high voltage source to maintain electrical charge on the plates of the capacitor.

[1] [1] page 73
[2] [7] page 416

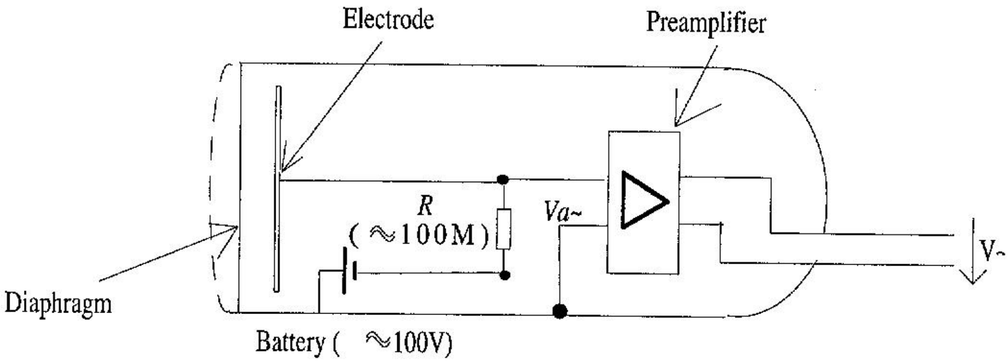


Fig. 2.19 Condenser microphone.

2.6.2 Contact microphones

Contact microphones (see Fig. 2.20) have the same basic principle as the crystal microphones. They consist of a small block of crystalline or ceramic material which is usually encased in metal or plastic. Two output wires are attached to two surfaces of the block. The block is attached to the body of an acoustic guitar, and works on the principle of the piezoelectric effect. When the strings are not vibrating, the body is at rest and the transducer generates no output. When a note is sounded, however, the movement of the vibrating body places the adjacent surface of the piezo block under compression or tension and generates an AC voltage. On the top of the block a mass is attached and due its sluggishness the mass has the effect of a force to the block. This force is proportional to the acceleration of the mass (Newton's law: $F=m*a$) and boosts the generated voltage. In view of the proportionality of the voltage to the force, the voltage is also proportional to the acceleration.

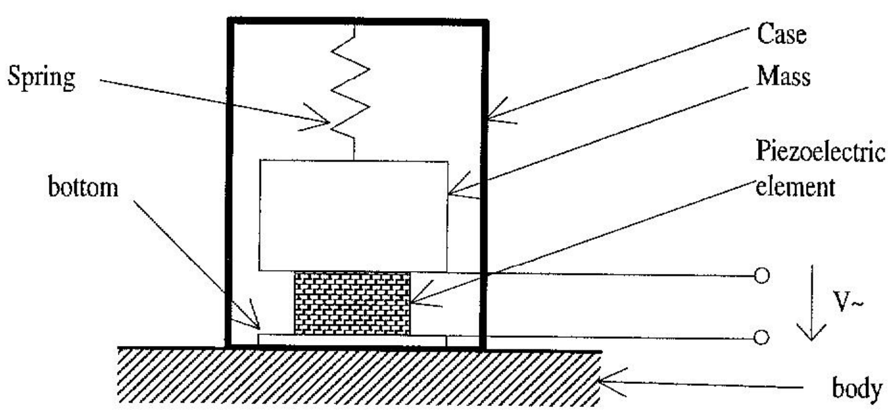


Fig. 2.20 Contact microphone.

The sensitivity of the contact microphone depends on the piezoelectric properties and on the size of the mass. The output voltage of the contact microphones is not very strong. Sometimes it is boosted by a preamplifier. Recent improvements in the design of piezoelectric crystals devices have resulted in a far more powerful output.

Contact microphones are fitted to the inside or the outside of the body. The principle of reacting to the vibrations of the body is common to all of them. The more centrally the microphone is placed on the body, the stronger the vibrations, therefore, the output voltage is stronger. [1]

[1] [1] page 79

2.6.3 Electric guitar pick-ups

The electromagnetic pick-ups consist of a coil with a magnetic core. The simplest form of magnetic pick-up consists of a permanent bar magnet with a continuous length of insulated copper wire wrapped around it several thousand times. This winding is an electrical coil. The thickness of the wire is extremely fine and its diameter of the copper wire is around 0.063 mm. [1]

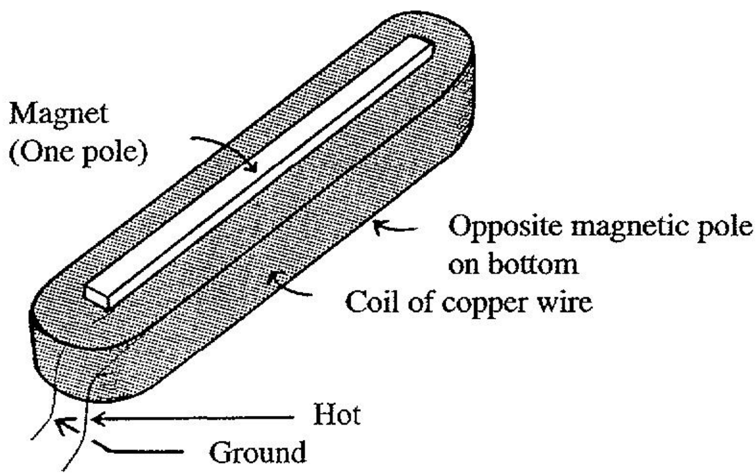


Fig. 2.21 The basic form of a pick up with a bar magnet.

The most famous single coil pick-up is the Fender Stratocaster pick-up (Fig. 2.22). There are several different variants of this pick-up, but they have something common. Instead of using a bar magnet, one employs six individual magnetic pole-pieces. A pole-piece acts to concentrate and direct the magnetic field so it is in an optimum shape and direction to sense the vibration of the strings. Pole-pieces can be of many shapes and sizes. The most applied pole-pieces are threaded and they can be adjusted in height by screwing it in or out. Adjusting the pole-pieces closer to the string will increase the volume. It can generate distortion if the string is too close to the magnet. This distortion will result due to the force exerted on the string by the magnet. All these six pole-pieces have the same magnetisation and usually the south pole is on the top close to the strings. The magnetic pole pieces are fixed with two plates of vulcanised fibre. These plates are composed of paper material saturated with resin and pressed under high pressure with the application of heat. The coil is actually wrapped around the pole-pieces. Originally it was made with 8350 turns of copper wire but newer Stratocaster pick-ups use only 7600 turns.

The dimension and the shape of the magnets as well the diameter and the turns of wire vary very often. In consideration of this it is very difficult to give exact values for the data of a Stratocaster pick-up. [2]

[1] [3] page 52
[2] [12] page 39

Newer models have the following data: [1]

Diameter of the magnet	: 4.75 mm
Length of the magnet pole pieces	: 16.70 mm
Distance between the magnets	: 10.34 mm (middle to middle)
Diameter of the copper wire	: 0.063 mm
Turns of wire	: ca 7600

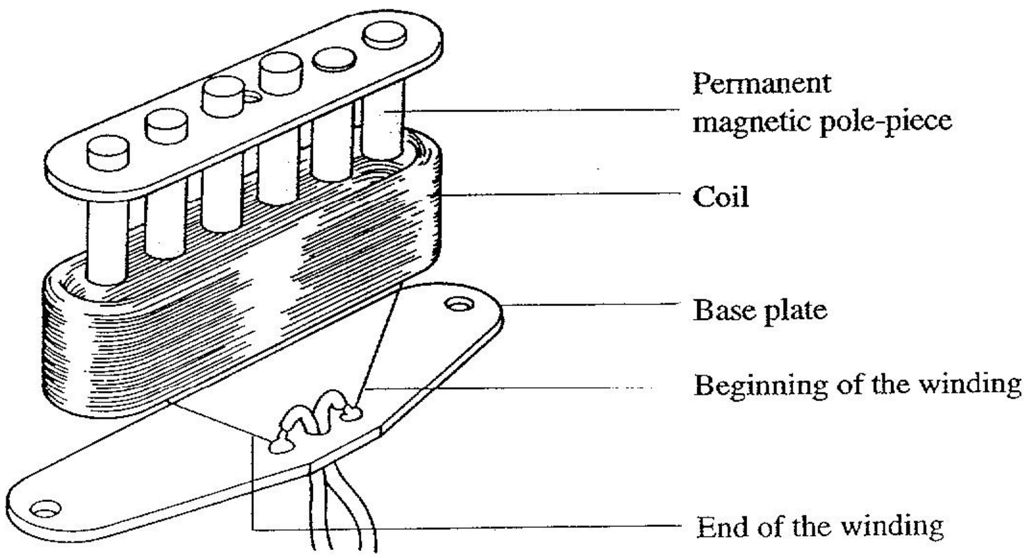


Fig. 2.22 The Fender Stratocaster pick-up.

2.6.3.1 The material of the magnets

A magnet is made by subjecting suitable material to a strong current which pulls all elementary magnets in one direction. Ordinary iron has its magnetic forces evenly mixed but applied current can change this condition. Pure iron does not make the most powerful magnets because the mixed forces in iron cannot retain the alignment as effectively as other metals. Most pick-up magnets are made of Alnico, an alloy of aluminium, nickel and cobalt. Its properties are more conducive to magnetisation than any other material. The formula of Alnico 5 is the most famous. Alnico 5 is composed of 8 parts aluminium, 14 parts nickel, 20 parts cobalt, 3 parts copper, and the remainder iron. There are many alnico formulas and each formula produces differently magnetic properties. The replacement of an alnico 5 magnet by an alnico 8 magnet will create a noticeable increase in treble response. Some manufacturers employ alternative ceramic or piezo magnets. A ceramic magnet increases treble responses even more than an alnico 8 magnet. Ceramic magnet pick-ups are generally more sensitive than alnico magnet pick-ups. This results in greater output and more trebles. [2]

[1] [1] page 92
[2] [12] page 17

2.6.3.2 Coil material

It is not possible to use bare wire for the coil because this would cause a short circuit and the wire needs to conduct like one very long wire. The wire for pick-ups is composed of solid copper which is a rather weak material. Coil wire is also made from aluminium, gold, as well as other metals, but copper is the most practical for pick-ups due to low cost and availability. It is coated with a poly-synthetic material or some other insulating substance to prevent conduction. A few years ago the standard coating on wire was lacquer, but lacquer tends to chip and crack. For the winding of the coil a small diameter wire is used which allows for a small coil to be tightly wrapped around a magnet. The diameter of the wire most commonly used is 42 AWG (American wire gauge: 42 AWG=0.06349mm). The coil of pick-ups is made with wire in the range from 36 AWG (0.127mm) to 54 AWG (0.01574mm). The finer the wire, the more sensitive a pick-up will be. The price of the wire rises when the diameter gets smaller.

It is important to take care when the wire is sharply bent around the end of magnets one and six, because if the insulation is broken, oxidation and crystallisation can occur. The wound wire is immersed in a hot wax bath to saturate the coil. This reduces microphonics and makes the coil more solid and durable. [1]

2.6.3.3 How the single coil pick-up works

In principle magnetic pick-ups are related to dynamic microphones. Both use electromagnetic forces. A dynamic microphone is coupled to a sound source through vibrating air and the moving air causes the microphone element to move. A magnetic pick-up acquires magnetic motion. It is a changing magnetic environment that results in an output of alternating voltage.

The magnet generates a magnetic static field around itself (Fig. 2.23a) and the guitar strings pass through the magnetic field. The strings are made of steel and they interact with the magnetic field. While the strings do not move, the field maintains a regular shape (Fig. 2.23b). The magnetic lines of force are more intense between string and magnetic south pole due the ferromagnetic behaviour of the string. The string pulls up the whole magnetic field and therefore the magnetic field also intensifies at the north pole.

As soon as a string is plucked its movement alters the shape of the magnetic field. When the string moves down to the magnet the lines of force start to move (Fig. 2.23c). When the string moves back, the lines of force start to move back (Fig. 2.23d). The vibrating string pulls and pushes against the invisible magnetic force of the magnet. The magnetic field moves according to the vibration of the string. [2]

[1] [12] page 18, 19
[2] [3] page 52

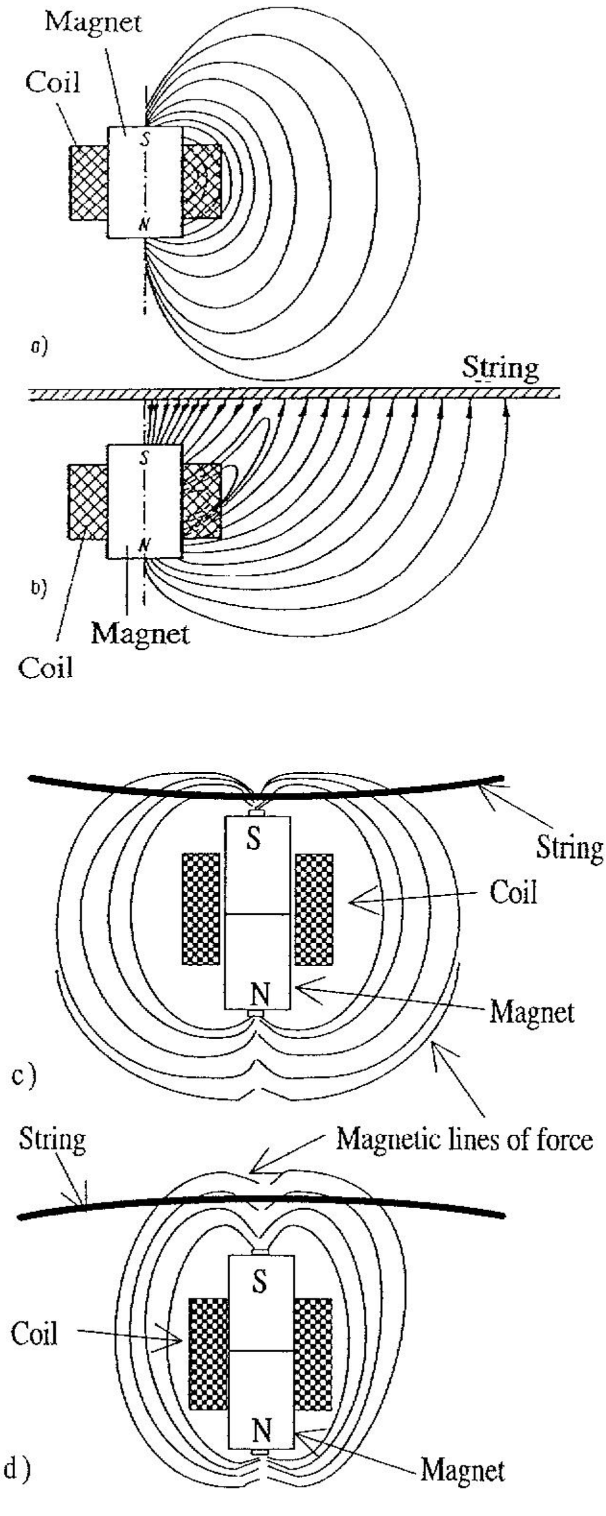


Fig. 2.23 How a string effects the magnetic flux of a pick-up.

The lines of force which make up the magnetic flux Φ intersect the coil. The vibrating string causes a change of the magnetic flux and it induces a voltage u_0 into the coil according to the change of Φ.

The value of the induced voltage u_0 can be calculated with the induction law:

$$u_0 = -N\frac{d\Phi}{dt}. \quad \textbf{Eq. (2.5-1)}$$

N = turns of wire in the coil

The amplitude u_0 of the induced voltage is proportional to the velocity v of the string vibration. Under the assumption of a pure sinusoidal vibration, the vibration position a_i of the string can be written as:

$$a_i = A_i \sin(2\pi f t). \quad \textbf{Eq. (2.5-2)}$$

A_i is the amplitude of the string vibration

f is the frequency

The velocity v can be expressed as:

$$v = \frac{da_i}{dt} = A_i 2\pi f \cos(2\pi f t). \quad \textbf{Eq. (2.5-3)}$$

The value of the induced voltage depends on the Amplitude A_i and on the frequency f of the string vibration. In additional there are many factors that influence the induced voltage u_0. These factors are: [1]

1) The strength of the magnetic field.
2) The number of turns of the coil.
3) The ferromagnetic properties of the string (i.e. permeability and cross-section of the string).
4) The plane of the vibration. The pick-up is more sensitive to vibration perpendicular to the magnetic pole as to vibration parallel to the magnetic pole.
5) The distance between the string and the magnetic pole.
6) The location and the geometry of the coil.
7) A further effect is generated due to the hard magnetic material of the string. A part of the string will be magnetised and then the string is itself a magnet. The effect is a more sensitive pick-up. The sensitivity depends on the strength of magnetisation.

Electromagnetic pick-ups generate in practice a voltage u_0 on a scale of 10-100 mV eff.

[1] [11] page 25

2.6.3.4 Humbucking pick-ups [1]

All pick-up coils are sensitive to interference from electromagnetic radiation. This means they can act as an antenna and pick up stray magnetic fields of amplifiers, fluorescent lights, 50 cycle hum or other electrical appliances when they are nearby. The humbucking pick-up (Fig. 2.24) was designed to eliminate this problem.

Humbucking pick-ups have two coils instead of one and they are combined out-of-phase. The stray magnetic field induces a hum-voltage u_{hum} in both coils, but because of the out-of-phase linkage the resulting hum-voltage u_{hum} is zero. To guarantee that the two coils do not cancel out the voltage generated by the vibrating strings as well, the sets of pole pieces within each coil have opposite magnetic polarities. This means that the top of one coil is magnetically north and the top of the other coil is magnetically south. The opposite magnetic polarity converts any signal magnetically sensed, back into an in-phase signal.

In summary a humbucking pick-up acts in the following manner:
Any signal (i.e. hum) sensed by the coils is cancelled, and any signal (i.e. string vibrations) sensed by the magnetic poles is accepted.

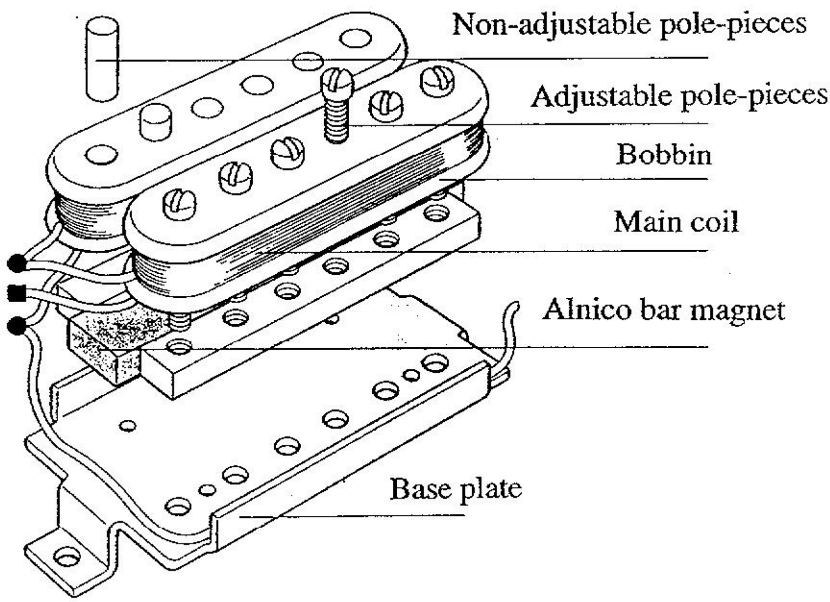


Fig. 2.24 Humbucking pick-up.

Traditionally, the two coils of a humbucking pick-up are linked together in a series circuit (Fig. 2.25a). The coils can also be linked in a parallel circuit (Fig. 2.25b). The manner of linking influences the electrical behaviour of the pick-up. When two equal coils are linked in series, the resultant resistance/inductance is the sum of the two individual resistances/inductances. When two resistances/inductances are linked in parallel the resultant resistance/inductance is one half of the sum.

[1] [12] page 21 and [3] page 53

There are some drawbacks to both, series and parallel linkages. In series linkage, there is a loss of high frequencies, and it is almost impossible to achieve an overall sound that is clear and delicate. In parallel linkage, the output level is considerably reduced, and it is virtually impossible to create a solid, beefy sound.

Series and parallel linkages have their own distinctive sound. The series sound is characterised by high volume with a good degree of bass and a favourable signal-to-noise ratio. A parallel sound is characterised by less volume, very bright and clear trebles, and a less favourable signal-to-noise ratio.

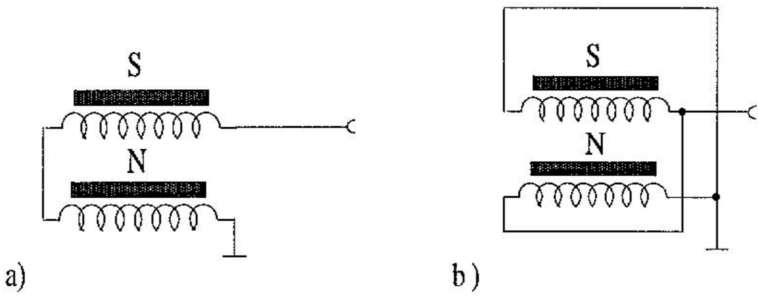


Fig. 2.25 The linkage of humbucking pick-ups.
a) Series linked.
b) Parallel linked.

2.6.3.5 Location of pick-ups [1]

The location of a pick-up affects the tone as well as the overall sound. One has the freedom to locate the pick-up between the bridge and the fretboard. When a pick-up is very close to the bridge, bass tones are greatly reduced. Pick-ups that are closer to the end of the fretboard than the bridge will give a fuller, less treble sound. Most electric guitars have at least two pick-ups, some have three. These pick-ups are located at different points along the string and they can be distinguished by different timbre.

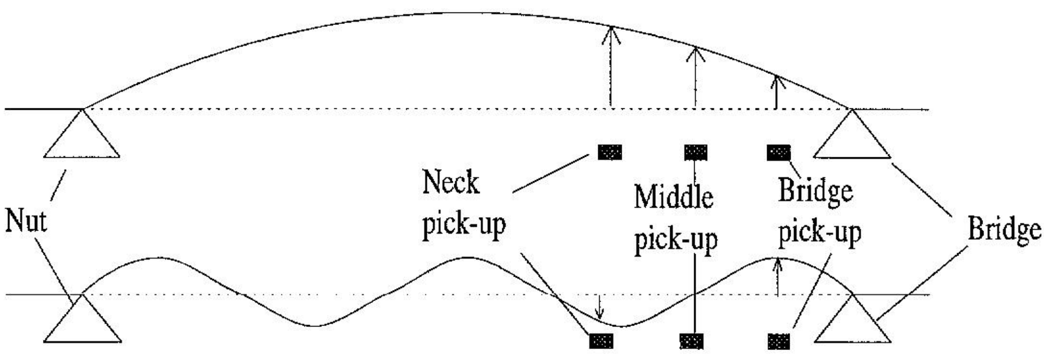


Fig. 2.26 Fundamental and 5th harmonic.

[1] [1] page 55-57

The distinctive timbres of differently located pick-ups are determined by the standing waves on the string (see section 2.5). If the pick-up is located at a place where the harmonic has a node then the harmonic will not generate a voltage into the coil. The harmonic is missing in the spectrum. The middle pick-up in Fig. 2.26 cannot sense the 5th harmonic because it is located directly under a node. In the spectrum of this pick-up is missing every multiple of the 5th harmonic. Low frequencies (such as the fundamental) induce a stronger voltage into the neck pick-up in view of the higher amplitude.

Numerical example:

The fundamental of the A string is 110 Hz. The middle of the pick-up is located at a distance of 16 cm to the bridge and the string length is 64 cm. With these assumptions the frequencies 440 Hz, 880 Hz, 1320 Hz ... are missing in the spectrum. The harmonics 220 Hz, 660 Hz, 1100 Hz ... have an antinode at the position of the pick-up and they induce maximum voltage. The transmission factor $\ddot{u}$ is defined as the ratio of the amplitude at the pick-up A_{pick} and the amplitude at an antinode $A_{antinode}$.

$$\ddot{u} = \frac{A_{pick}}{A_{antinode}}. \qquad \textbf{Eq. (2.6-1)}$$

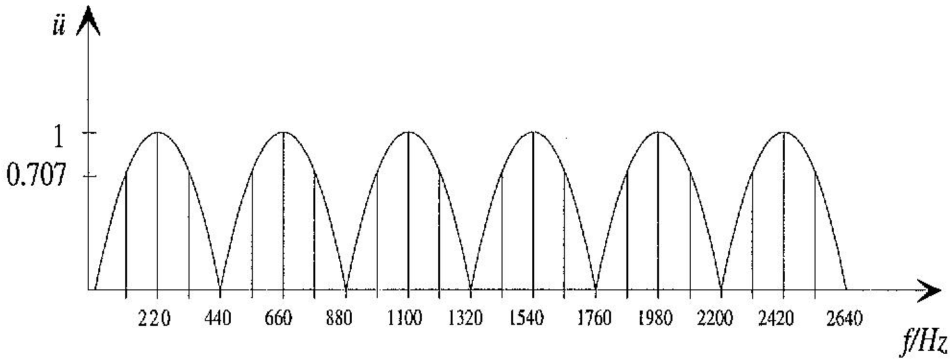


Fig. 2.27 Transmission diagram for a pick-up 16 cm away from the bridge (A string, l=64 cm).

This knowledge is exploited at the bridge pick-ups on Telecaster and Stratocaster guitars. They are angled so that the treble side of the pick-up is closer to the bridge than the bass side. If the bass side would be as close to the bridge as the treble side, the bass would be very thin and weak.

With two or three pick-ups located at different places it is possible to reach different timbres with the same guitar. Switches or individual gain controls allow the guitarist to mix the signals together from the pick-ups as desired.

3. Measurements and studies

3.1 The equivalent circuit diagram of the pick-up

The exterior of pick-ups is sometimes very similar but their sound is quite different. It is essential for the understanding to get familiar with the AC characteristic of pick-ups. For the further studies it is necessary to consider the equivalent circuit diagram of the air-core inductor (Fig. 3.1). It consists of the inductance L, the DC resistance R and the winding capacitance C_W. The value of C_W can be a few hundred pF. The value is influenced by the geometry of the coil, the thickness of the varnish insulation and the manner of winding. A tight winding and a parallel direction of the wire increase the value of C_W. The winding capacitance C_W effects in the same way as a parallel connected capacitance. There exists also a capacitance to ground C_G. If the coil gets a case then a capacitance C_E is added.

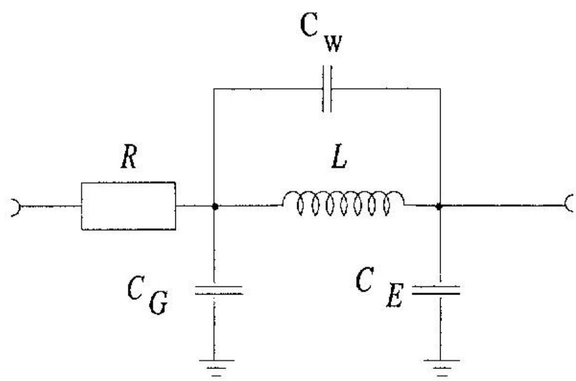


***Fig. 3.1** The equivalent circuit diagram of the air-core inductor.*

The decisive component is the inductance L. It depends on the number of turns per unit length and the geometry of the coil. In the case of pick-ups it depends also on the dimension and properties of the iron core and of the magnets. The most pick-ups have an inductance of a few Henry [H]. The capacitances C_W, C_G and C_E can be combined in a total capacitance which is parallel to the inductance L.

It is found that a good equivalent circuit diagram for a pick-up is the one shown in Fig. 3.2 ([1] page 87).

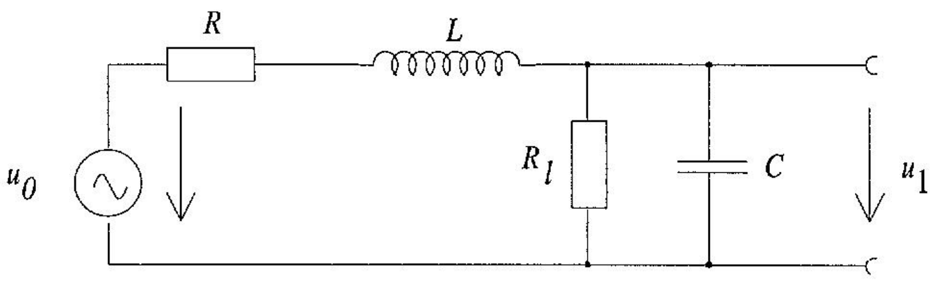


Fig. 3.2 Equivalent circuit diagram of the pick-up.

The capacitance C stands for the total capacitance of the pick-up. The losses of the pick-up are considered with a corresponding loss resistance R_l. Particularly losses appear as a result of eddy-current, hysteresis and remanence losses in the iron. Further losses originate in the coil and as well scattering losses are added. The voltage u_0 is the source voltage and it is induced due to the string vibration. The output voltage, at the connections of the pick-up without any external load, is u_1.

3.1.1 Theoretical calculation of the impedance

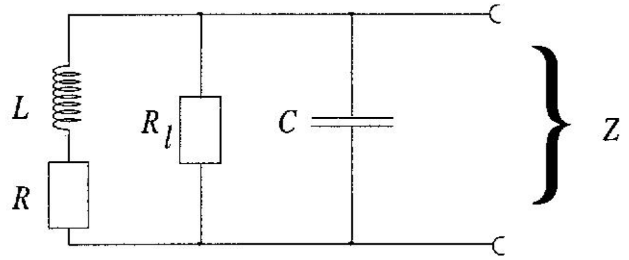


Fig. 3.3 The complex frequency-dependent impedance Z.

The inductance L and the resistance R are linked in series and they are connected with R_l and C in parallel. The equation for Z can be written as:

$$Z = \frac{1}{\frac{1}{R + j\omega L} + \frac{1}{R_l} + j\omega C}. \qquad \textbf{Eq. (3.1-1)}$$

This equation is not divided in real and imaginary part. For further applications it is necessary to apply mathematical transformations to get a clear equation. After a few transformations Eq. (3.1-1) can be expressed as:

$$Z = R \cdot \frac{1 + j\omega \frac{L}{R}}{1 + \frac{R}{R_l} - \omega^2 CL + j\omega\left(\frac{L}{R_l} + RC\right)}. \qquad \textbf{Eq. (3.1-2)}$$

The denominator and numerator are divided in a real and an imaginary part. Some time constants and the self-resonant frequency of the pick-up can be substituted in this equation.

The following variables are substituted:

$K = R/R_l$ = constant.

$T_1 = L/R$ = time constant 1.

$T_2 = L/R_l$+RC = time constant 2.

$\omega_0{}^2 = 1/LC$ = self-resonant radian frequency of the pick-up squared.

With these replacements the equation can be written as:

$$Z = R \cdot \frac{1 + j\omega T_1}{1 + K - \left(\frac{\omega}{\omega_0}\right)^2 + j\omega T_2}. \qquad \textbf{Eq. (3.1-3)}$$

The absolute value of the impedance Z is significant for further investigations. It determines the response characteristic of the unloaded pick-up. The absolute value can be calculated by the division with the absolute value of the denominator and the absolute value of the numerator. The result is:

$$|Z| = R \cdot \frac{\sqrt{1 + (\omega T_1)^2}}{\sqrt{\left(1 + K - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + (\omega T_2)^2}}. \qquad \textbf{Eq. (3.1-4)}$$

The typical frequency dependence of the absolute value of impedance is shown in Fig. 3.4. Pick-ups have usually a distinctive maximum in the range from 5 kHz to 15 kHz. At very low frequencies the DC resistance is dominant and the absolute value of the impedance corresponds to R. If the frequency increases then the impedance rises according to the inductance L. At a specified frequency f_0 the impedance reaches the maximum value. This is the resonance frequency of the parallel circuit. At higher frequencies the impedance decreases in view of the capacitance C.

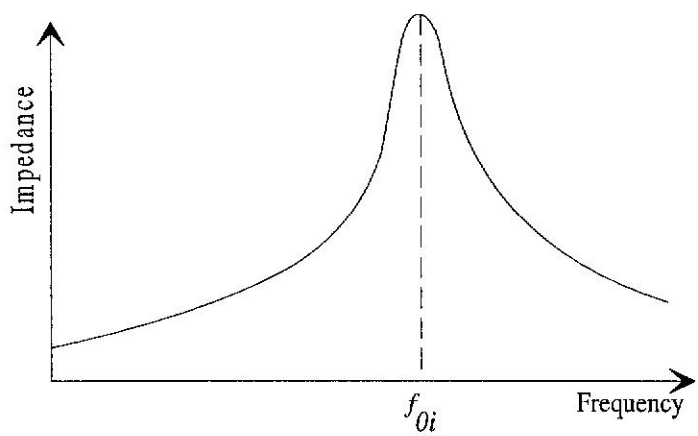


Fig. 3.4 Typical frequency dependence of the impedance (magnitude).

The phase can be calculated by a subtraction of the denominator phase and the numerator phase of Eq. (3.1-3). The result is:

$$\varphi_p = \arctan(\omega T_1) - \arctan\left(\frac{\omega T_2}{1 + K - \left(\frac{\omega}{\omega_0}\right)^2}\right). \qquad \textbf{Eq. (3.1-5)}$$

The pick-up effects inductively at low frequencies and it effects capacitively at frequencies higher than the resonance frequency. The curve of the phase is shown in the following figure.

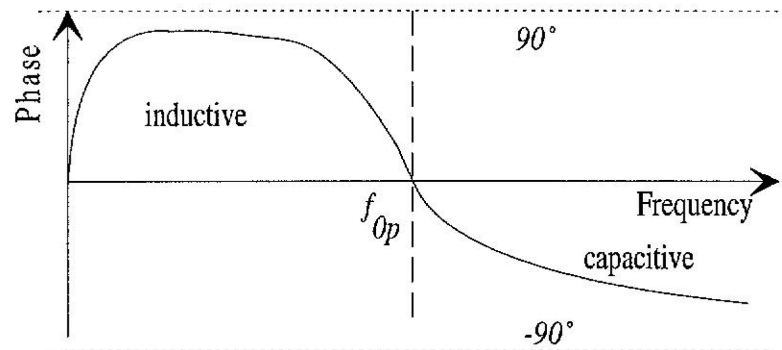


Fig. 3.5 Frequency dependence of the phase of the impedance.

It is necessary to mention that the resonance frequency of the phase ($\varphi_{p(f_{0p})} = 0°$) is not the same as the resonance frequency of the absolute impedance ($|Z|_{f_{0i}}$ *=max.*). The difference is only a few Hz. In the next sections it is assumed that the resonance frequency can be calculated with the self-resonant frequency of a parallel circuit: [1]

$$f_0 = \frac{1}{2\pi\sqrt{LC}}. \qquad \textbf{Eq. (3.1-6)}$$

The accuracy of this assumption is good enough for further studies with the pick-up. The result f_0 of this equation is different from f_{0i} and f_{0p} and usually the value of f_0 is in the range from f_{0i} to f_{0p}.

[1] [1] page 87

3.2 DC resistance *R*

The DC resistance is determined by several factors. It depends on the length of wire l_w, the wire cross section A_w, and on the resistivity δ_w of the wire material. The wire material is mostly copper but sometimes it can be gold or aluminium. It is difficult to get the same DC resistance for two pick-ups because small fluctuations in the cross section are noticeable in the resistance. The DC resistance can be measured easily with an ohmmeter. For the theoretical calculation the following equation is valid: [1]

$$R = \frac{\delta_w l_w}{A_w}. \qquad \textbf{Eq. (3.2-1)}$$

Generally the DC resistance of pick-ups is in a range from 5 kΩ to 15 kΩ. A higher DC resistance strangles the output voltage of the pick-up.
All DC resistance measurements were done with the following equipment.

PM 6304 programmable automatic RCL meter (Fluke and Philips).

For the measurements 25 pick-ups were at one's disposal. 15 pick-ups were of the same type and the task was to compare the results. The interest was how uniform the production process of the pick-ups is. The remaining 10 pick-ups were from total different types and vintages, and the task here was to find out these differences.

3.2.1 DC resistance of *Stratocaster* pick-ups

The type of the 15 pick-ups:
SSL-1 Vintage Staggered Pick-Up for Stratocaster guitar (Seymor Duncan).
The results of the measurements are depected in the following table 3.1.

SSL-1 Pick-ups					
Nr.	resistance/[kΩ]	Nr.	resistance/[kΩ]	Nr.	resistance/[kΩ]
1	6.88	6	6.21	11	6.19
2	6.23	7	6.49	12	6.71
3	6.67	8	6.32	13	6.81
4	6.53	9	6.52	14	6.27
5	6.56	10	6.35	15	6.38

Table 3.1 Measured resistances of the SSL-1 Pick-ups.

[1] [7] page 399

The DC resistances of the 15 pick-ups are similar and the fluctuations are relatively small. The differences exist due to fluctuations in the cross section or the length of the wire (Eq. (3.2-1)). For the analyses the equations for the mean value and the standard deviation were used. The equations are given in Eq. (3.2-2) and Eq. (3.2-3).

$$\overline{x} = \sum_{i=1}^{n} \frac{x_i}{n}. \qquad \textbf{Eq. (3.2-2)}$$

$$s = \sqrt{\frac{1}{1-n} \sum_{i=1}^{n} (x_i - \overline{x})^2}. \qquad \textbf{Eq. (3.2-3)}$$

$\overline{x}$ = mean value
x_i = value number
n = number of data
s = standard deviation

Some measurement results:

Maximum value	: $R_{max} = 6.88$ kΩ
Minimum value	: $R_{min} = 6.19$ kΩ
Mean value	: $R_m = 6.47$ kΩ
Standard deviation	: $R_s = 221$ Ω

3.2.2 DC resistance of other pick-ups

The results of the measurements are in the following table.

Different types of pick-ups	DC resistance/[kΩ]
De Armond/Dynasonic 1957 Gretsch 1	9.63
De Armond/Dynasonic 1957 Gretsch 2	7.25
voice pick-up proto 1	7.35
voice pick-up proto 2	9.65
voice pick-up proto 3	5.72
Fender Lace Sensor 1	5.68
Fender Lace Sensor 2	14.56
Seymour Duncan P-90 Soapbar	7.80
Seymour Duncan Humbucker Jeff-Beck Model	4.14
Fender Humbucker	10.62

Table 3.2 Measured DC resistances of the different pick-ups.

Three pick ups in table 3.2 are remarkable. These are the *Fender Lace Sensor 2* with a considerably high resistance, the *Fender Humbucker* with a relatively high resistance and the *Seymor Duncan humbucker* with a very low resistance. High resistances are indicating either single coil pick-ups with a very long wire (as the *Fender Lace Sensor 2*) or humbucker pick-ups with a series connection of the two pick-ups (as the *Fender Humbucker*). It is obvious that the *Seymor Duncan Humbucker* must be connected in a parallel configuration in order to have a low resistance. The rest of the pick-ups have a DC resistance in the expected range from 5 kΩ to 10 kΩ.

3.3 Inductance *L*

For further investigations the AC characteristic of the pick-up is important and due to this the next step is the determination of the inductance *L*. The inductance depends on the number of turns per unit length, the geometry of the coil and the amount of iron in the coil interior (see section 3.1). It is very difficult to determine the inductance of a pick-up by theoretical calculations. The reasons are the shape of the coil and the relative permeability of the coil interior. The determination of the inductance by measurements is the best way to get exact results.

The determination was done by two different measurements. One solution is the measurement with appropriate equipment. With the "*PM 6304 programmable automatic RCL meter*" it is possible to measure the inductance *L*.

It is possible to determine the inductance *L* by using a shunt resistance R_{sh} for the measurement. This measurement was done at a low frequency. For low frequencies the impedance is determined by the resistance *R* and the inductance *L*. Fig. 3.6 shows the measuring arrangement:

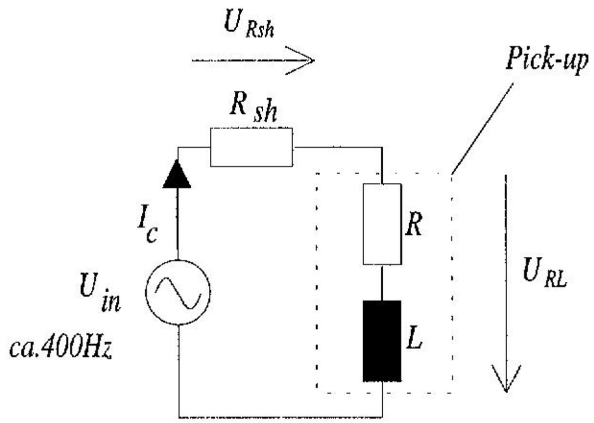


Fig. 3.6 Measuring arrangement for low frequencies.

For the calculation of the inductance L it is necessary to measure the voltage U_{Rsh} and the voltage U_{RL}. The inductance L can be calculated by the following equation:

$$L = \frac{1}{2\pi f} \cdot \sqrt{\left(\frac{U_{RL}}{U_{Rsh}} R_{sh}\right)^2 - R^2} \; . \qquad \textbf{Eq. (3.3-1)}$$

The results of both measurements are very similar. For this reason the results of both measurements are listed as a mean value in table 3.3 and table 3.4.

3.3.1 Inductance of *Stratocaster* pick-ups

The results for the inductance L of the *Stratocaster* pick-ups are depicted in the following table:

SSL-1 Pick-ups					
Nr.	inductance L/[H]	Nr.	inductance L/[H]	Nr.	inductance L/[H]
1	2.76	6	2.75	11	2.71
2	2.70	7	2.68	12	2.79
3	2.70	8	2.74	13	2.69
4	2.72	9	2.63	14	2.71
5	2.75	10	2.69	15	2.43

Table 3.3 Measured inductances of the SSL-1 Pick-ups.

The inductances of the 15 pick-ups are very similar. Only the pick-up Nr.15 has a conspicuously small inductance L. This is a sign of poor production quality of this pick-up. There could be some problems with the winding or with the relative permeability of the material interior.

Some measurement results:

Maximum value	: $L_{max} = 2.79$ H
Minimum value	: $L_{min} = 2.43$ H
Mean value	: $L_m = 2.70$ H
Standard deviation	: $L_s = 83.4$ mH

3.3.2 Inductance of other pick-ups

The results for the inductance L are given in the following table:

Different types of pick-ups	inductance L/[H]
De Armond/Dynasonic 1957 Gretsch 1	3.83
De Armond/Dynasonic 1957 Gretsch 2	2.04
voice pick-up proto 1	2.10
voice pick-up proto 2	3.46
voice pick-up proto 3	1.38
Fender Lace Sensor 1	2.24
Fender Lace Sensor 2	7.85
Seymour Duncan P-90 Soapbar	7.04
Seymour Duncan Humbucker Jeff-Beck Model	2.12
Fender Humbucker	4.80

Table 3.4 Measured inductances of the different pick-ups.

Usually the inductance of pick-ups is in the range from 1 H to 10 H. This assumption was confirmed of the available pick-ups. Remarkable are the *Fender Lace Sensor 1* and the *Seymor Duncan P-90*. Both have a high inductance. The former pick-up has also a high resistance and this indicates a very long wire with more turns than the other pick-ups. The latter pick-up must have a material with higher relative permeability because its resistance does not indicate a high inductance. The *voice pick-up proto 3* has a quite low inductance but this was intended to create new sounds (The *voice pick-ups* are homemade pick-ups).

3.4 Resonance frequency f_0 and capacitance C

The decisive factor for the sound of a pick-up is the resonance frequency f_0. It decides if the sound series is mellow, brilliant, shrill, or harsh. The transfer function relates directly to the frequency dependence of the impedance (section 3.7.3). For this reason it is important to determine the resonance frequency f_0 of the impedance. The measuring arrangement should be realised with very short cables. This is important to minimise the influence of the cable capacitance otherwise it would falsify the real resonance frequency of the pick-up.

Another reason is the determination of the capacitance C through the resonance frequency. The result would be the sum of the pick-up capacitance and the cable capacitance. The equation of the capacitance C can be derived by Eq. (3.1-6).

$$C = \frac{1}{(2\pi f_0)^2 L}. \qquad \textbf{Eq. (3.3-2)}$$

The influence of the voltmeter is minimised by a parallel connection with a shunt resistance. The resonance frequency can be measured by changing the frequency at the generator. The voltage U_{Rs} reaches its minimum at the frequency f_0.

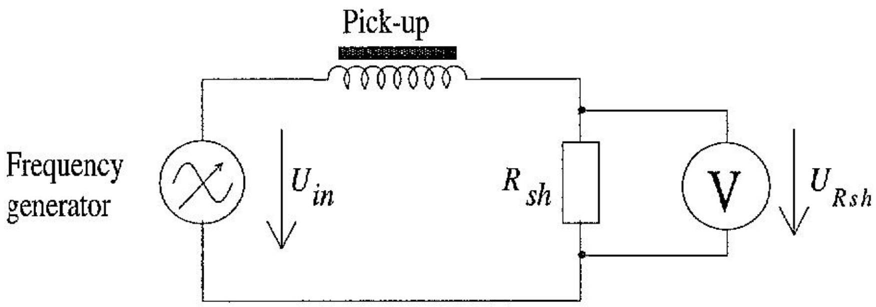


Fig. 3.7 Measuring arrangement for the resonance frequency.

3.4.1 Resonance frequency and capacitance of *Stratocaster* pick-ups

SSL-1 pick-ups								
Nr.	f_0/[kHz]	C/[pF]	Nr.	f_0/[kHz]	C/[pF]	Nr.	f_0/[kHz]	C/[pF]
1	10.71	79.7	6	11.14	74.2	11	11.14	75.3
2	11.23	74.4	7	10.91	79.4	12	10.36	84.6
3	10.56	84.1	8	10.90	77.8	13	10.85	80.0
4	10.79	80.0	9	9.74	101.5	14	11.22	74.2
5	10.71	80.3	10	10.97	78.2	15	10.40	80.2

Table 3.5 Measured resonance frequencies f_0 and calculated capacitances C of the 15 SSL-1 pick-ups.

The measuring device should have a very high input resistance and a very low input capacitance to minimise the influence. The shunt resistance should be in the scale of $1\ k\Omega$.

In table 3.5 pick-up Nr. 9 is remarkable because of a relatively high capacitance and a low resonance frequency. The reason could be a thinner insulation because it would cause a higher capacitance and a lower resonance frequency.

Some measurement results:

Maximum values	: f_{0max} = 11.23 kHz	, C_{max} = 101.5 pF
Minimum values	: f_{0min} = 9.74 kHz	, C_{min} = 74.2 pF
Mean values	: f_{0m} = 10.78 kHz	, C_m = 80.3 pF
Standard deviations	: f_{0s} = 395 Hz	, C_s = 6.7 pF

3.4.2 Resonance frequency and capacitance of other pick-ups

Different types of pick-ups	f_0 / [kHz]	C / [pF]
De Armond/Dynasonic1957 Gretsch 1	13.27	37.6
De Armond/Dynasonic1957 Gretsch 2	24.05	21.5
voice pick-up proto 1	18.69	34.5
voice pick-up proto 2	15.45	30.7
voice pick-up proto 3	21.90	38.3
Fender Lace Sensor 1	8.16	169.8
Fender Lace Sensor 2	3.08	340
Seymor Duncan P-90 Soapbar	5.21	132.6
Seymor Duncan Humbucker J.-B. Model	8.69	158
Fender Humbucker	7.27	99.8

Table 3.6 Measured resonance frequencies f_0 and calculated capacitances C of the different pick-up types.

The conspicuously unlike resonance frequencies in table 3.6 are remarkable. It is obvious that every pick-up has to have its own distinctive sound. The *Fender Lace Sensor 2* has an extremely low resonance frequency and a very high capacitance. The *De Armond Dynasonic 1957 Gretsch 2* has the opposite behaviour with a distinctively high resonance frequency and a very small capacitance. All other pick-ups have their resonance frequency and their capacitance in the scale between the mentioned pick-ups. It is important to notice the influence of guitar cable capacitances and input resistances on the resonance frequency. The explanation of these effects is described in section 3.8.

3.5 Absolute value of impedance and loss resistance

For the determination of the loss resistance R_l it is necessary to measure the absolute value of the impedance $|Z|$. This can be done with the same measuring arrangement as shown in Fig. 3.7. In section 3.1.1 it was already shown that the phase of the impedance at the resonance frequency is 0 °. In view of this, the impedance of the pick-up has only a real part. The impedance at the resonance frequency can be calculated by the following equation:

$$|Z| = \frac{U_{Pick-up}}{U_{Rsh}} \cdot R_{sh} \,. \qquad \textbf{Eq. (3.4-1)}$$

The calculation of the loss resistance R_l can be derived from Eq. (3.1-4). With several mathematical transformations a solution is found for R_l. The most important transformations are shown on this page. The first equation is directly derived from Eq. (3.1-4).

$$\left(1+K-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + (\omega T_2)^2 = \frac{R^2\left(1+(\omega T_1)^2\right)}{|Z|^2} \,. \qquad \textbf{Eq. (3.4-2)}$$

For the calculation of R_l the radian frequency ω is set to be ω_0. This assumption simplifies the whole equation and it can be solved by R_l.

$$\left(\frac{R}{R_l}\right)^2 + \omega^2\left(\frac{L}{R_l}+RC\right)^2 = \frac{R^2\left(1+\left(\omega\frac{L}{R}\right)^2\right)}{|Z|^2} \,. \qquad \textbf{Eq. (3.4-3)}$$

When R_l is taken outside, the following quadratic equation is the result:

$$R_l^{\,2}\left((\omega RC)^2 - \frac{R^2\left(1+\left(\omega\frac{L}{R}\right)^2\right)}{|Z|^2}\right) + 2RR_l + R^2 + (\omega L)^2 = 0 \,. \qquad \textbf{Eq. (3.4-4)}$$

The solution for this quadratic equation will provide two solutions for the loss resistance R_l. Only one of both solutions is the correct value of R_l.

The correct solution for this quadratic equation is:

$$R_l = \frac{-R - \sqrt{R^2 - \left((\omega RC)^2 - \frac{R^2\left(1+\left(\omega \frac{L}{R}\right)^2\right)}{|Z|^2} \right) \cdot \left(R^2 + (\omega L)^2\right)}}{\left((\omega RC)^2 - \frac{R^2\left(1+\left(\omega \frac{L}{R}\right)^2\right)}{|Z|^2} \right)} . \quad \textbf{Eq. (3.4-5)}$$

3.5.1 Absolute value of impedance and loss resistance of *Stratocaster* pick-ups

SSL-1 pick-ups								
Nr.	$\|Z\|$ /kΩ	R_l/kΩ	Nr.	$\|Z\|$ /kΩ	R_l/kΩ	Nr.	$\|Z\|$ /kΩ	R_l/kΩ
1	1178	1539	6	1347	1739	11	1378	1806
2	1403	1848	7	1386	1889	12	1201	1589
3	1253	1693	8	1288	1675	13	1221	1621
4	1339	1802	9	1184	1686	14	1354	1764
5	1339	1800	10	1331	1764	15	1166	1545

Table 3.7 Measured absolute impedance $|Z|$ and calculated loss resistance R_l of the 15 SSL-1 pick-ups.

The pick-ups have in general a very high loss resistance R_l, therefore, the losses are small. The pick-ups Nr.1 and Nr.15 have a relatively small resonance impedance compared with the other pick-ups and in view of this the loss resistance is smaller.

Some measurement results:

Maximum values	: $\|Z\|_{max}$ = 1403 kΩ	, R_{lmax} = 1889 kΩ
Minimum values	: $\|Z\|_{min}$ = 1166 kΩ	, R_{lmin} = 1539 kΩ
Mean values	: $\|Z\|_m$ = 1291 kΩ	, R_{lm} = 1717 kΩ
Standard deviations	: $\|Z\|_s$ = 83.1 kΩ	, R_{ls} = 108.2 kΩ

3.5.2 Absolute value of impedance and loss resistance of other pick-ups

Different types of pick-ups	$\lvert Z \rvert$ / [kΩ]	R_l / [kΩ]
De Armond/Dynasonic1957 Gretsch 1	962	1058
De Armond/Dynasonic1957 Gretsch 2	1184	1302
voice pick-up proto 1	2241	3072
voice pick-up proto 2	2493	3168
voice pick-up proto 3	1645	2225
Fender Lace Sensor 1	446	552
Fender Lace Sensor 2	704	1261
Seymor Duncan P-90 Soapbar	494	533
Seymor Duncan Humbucker J.B. Model	303	334
Fender Humbucker	680	800

Table 3.8 Measured absolute impedance $\lvert Z \rvert$ and calculated loss resistance R_l of the different pick-up types.

The pick-ups have very different absolute values of impedance and in view of this they also have very different loss resistances. The conspicuously high impedance of the *voice-pick-up proto 2* is remarkable. It has the absolute highest impedance and loss resistance of all pick-ups. The opposite is the *Seymor Duncan Humbucker J.B. Model* with the smallest impedance and loss resistance. The loss resistance is a factor which determines the peak of the impedance at the resonance frequency. A pick-up with no distinctive peak at the resonance frequency will have a weak sound.

3.6 Theoretical calculations with MATLAB

For the numerical calculations based on the theory and equivalent circuits the MATLAB program was used. MATLAB is a technical computing environment for high-performance numeric computation and visualization. MATLAB integrates numerical analysis, matrix computation, signal processing, and graphics in an environment where problems and solutions are expressed just as they are written mathematically. For further information about MATLAB refer to the manuals. [1]

MATLAB was used to compute the following impedance properties:

1) the absolute value of impedance Z
2) the resistance, real(Z) (real part of the impedance)
3) the reactance, im(Z) (imaginary part of the impedance)
4) the phase φ_p of the impedance Z

[1] [17]

Five different pick-ups were selected for the computation. These are:

1) *SSL-1 Vintage pick-up for Stratocaster* guitar (*Seymor Duncan*). In this case the mean values of all 15 pick-ups were used.
2) *De Armond/Dynasonic 1957 Gretsch 1.*
3) *Voice pick-up proto 2.*
4) *Seymor Duncan P-90 Soapbar.*
5) *Fender Lace Sensor 2.*

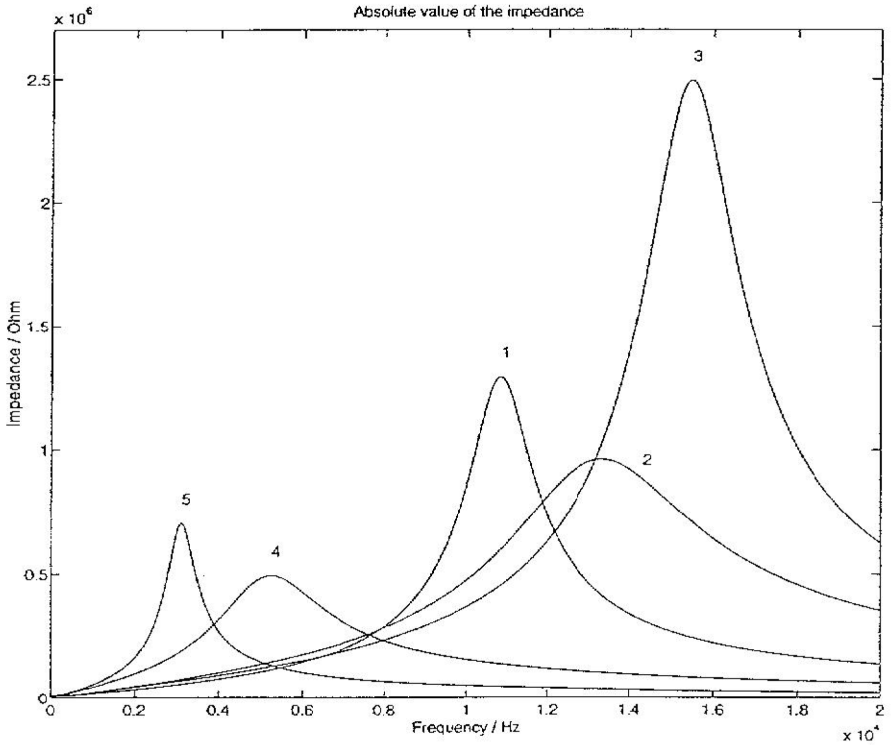


Fig. 3.8 Frequency curves of the absolute value of impedance for different pick-ups.

In this diagram the different frequency curves are represented. It is obvious that the whole impedance curves of all pick-ups are distinctively different, especially the various absolute values and the unlike resonance frequencies are outstanding.

The dependencies of the resistive and reactive components on the frequency are represented in Fig. 3.9 and Fig. 3.10. It is noticeable that the resistance curves have their peaks at the same frequencies. The peaks are identical for the absolute value of impedance. This is understandable in view of the reactance curves. They have the sign change from plus to minus at the resonance frequency. The reactance curves have two peaks with similar values only the sign is different. This shows the influence of the inductance L and the influence of the capacitance C.

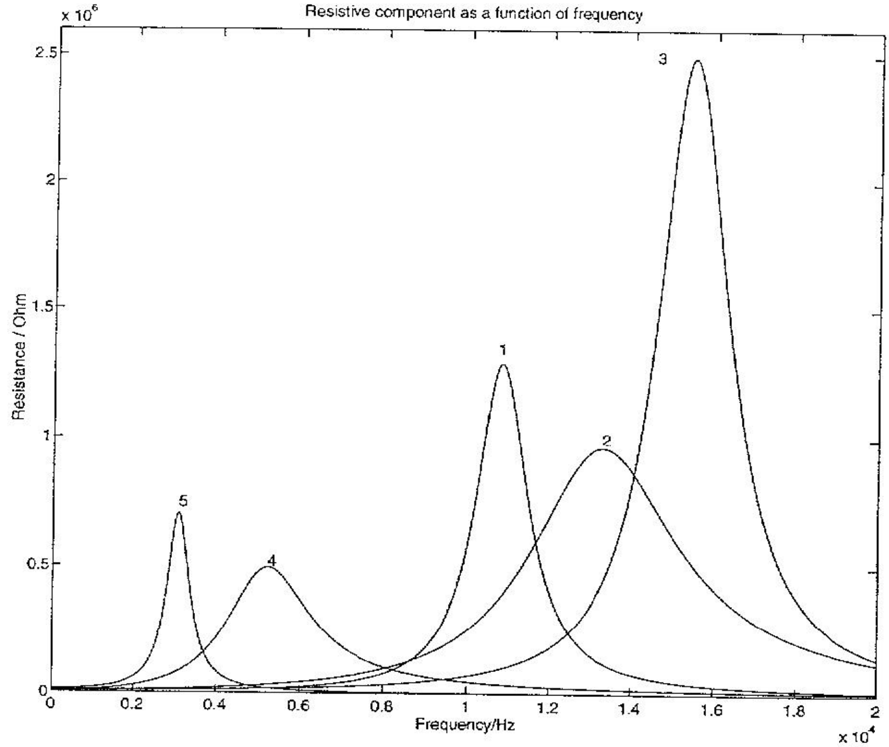


Fig. 3.9 Resistive component of different pick-ups.

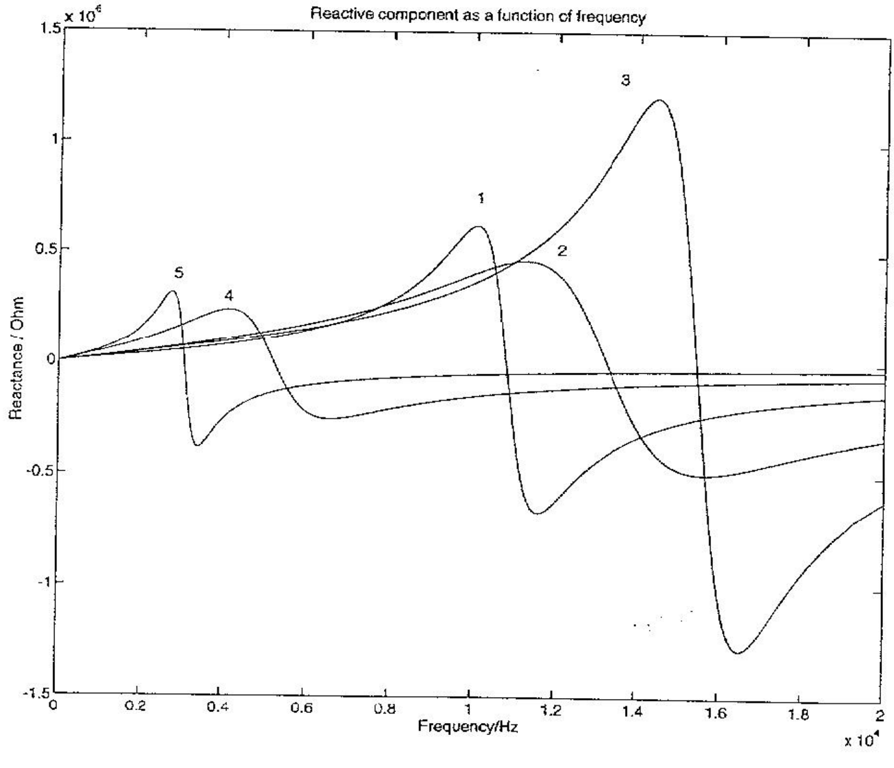


Fig. 3.10 Reactive component of different pick-ups.

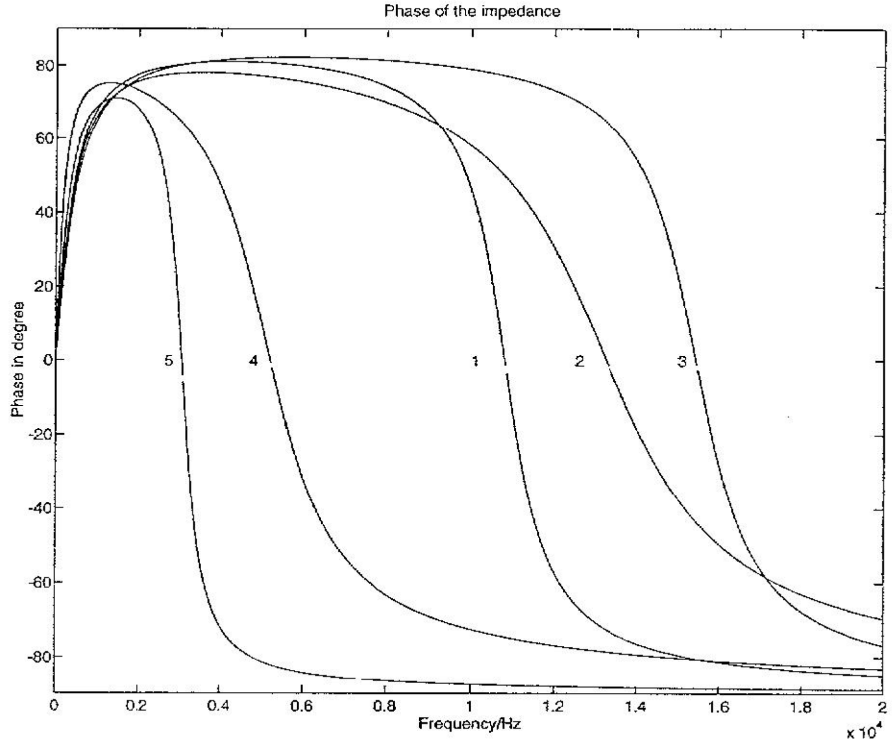


Fig. 3.11 Phase of different pick-ups.

This diagram shows that the phase curve changes its sign very fast in the range of the resonance frequency. It also confirms the effect of the impedance changing from inductance to capacitance.

All MATLAB programs for the diagrams in this section are listed in the Appendix B. The programs include comments for the commands so that it is possible to understand the programs.

3.7 Measurements with QuickSig

3.7.1 Short description of QuickSig [1]

QuickSig is a digital signal processing environment based on the latest developments in object-oriented programming (OOP). OOP is one of the most successful new paradims in making large and complex software systems possible. In several earlier tools for digital signal processing (DSP), OOP was already used. The idea of QuickSig was to stretch out the LISP language by a layer of general DSP constructs. These are abstract data structures like signals, filters, windows, graphical presentations, and related signal processing operations. QuickSig is a system which is aimed for algorithmic development and it is easily extendible to include new ways of modelling numerical and symbolic signals and signal processing. Conventional DSP is founded almost entirely on numeric computations using simple data structures like scalar numbers, arrays and specific file formats for signals, spectra, etc. The brightness of abstract concepts included in signal processing is not shown in this formalism. The higher abstraction levels and symbolic manipulations of signals depend on the mental processing of the programmer. They are not available as an integral part of the program or of the programming environment.

Advances in artificial intelligence (AI) and modern programming languages have given the possibility to find more powerful and systematic solutions to problems with the help of OOP. How to represent information and knowledge and how to solve problems were the main questions in AI. An Object-like formalism solves the question on representation. The solving of problems is done typically by rule-based and logic programming. OOP relates to AI languages such as Prolog and especially to LISP. The Common Lisp Object System (CLOS) is a portable and widely accepted OOP tool for LISP programmers. Other alternatives are Smalltalk and C++. C++ has become popular to make commercial programs for engineering purposes.

QuickSig is an experimental object oriented DSP programming environment and it is meant to be general and more engineering oriented than its predecessors. In speech processing studies the QuickSig system is extensively applied. QuickSig was originally written in Common Lisp and New Flavors, which is close to the CLOS standard. The QuickSig essence can be considered as a signal processing extension on top of the LISP language and CLOS object system.

Today an Apple Macintosh is used as the host computer. The main hardware features of the present QuickSig system for measurement purposes are:

Processor-card: NB-DSP2300 for NuBus (Macintosh).
Audio-card : NB-A2100 for NuBus (Macintosh).
A/D and D/A converter with 16 bit (Two channels in and out).
Sampling frequency is 44.1 kHz
For further information about QuickSig refer to [15] and [16].

[1] [15] and [16]

3.7.2 Measurements of the impedance with QuickSig

The idea to do the measurements with QuickSig is based on the theory of impulse response and transfer function. With the QuickSig system it is possible to measure the impulse response *h(t)* of a signal in the time domain. The principle is shown in the following picture.

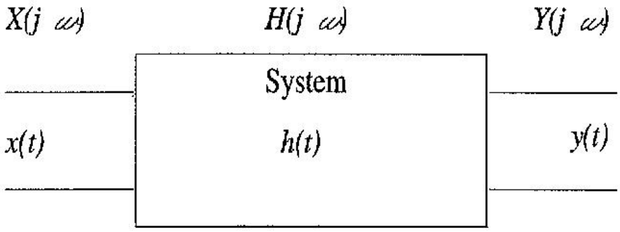


Fig. 3.12 A linear system, its input-output relationship, impulse response, and transfer function.

The QuickSig system generates an input signal *x(t)* and measures the output signal *y(t)*. The impulse response is linked with *x(t)* and *y(t)* through the convolution integral. [1]

$$y(t) = x(t) * h(t) = \int_0^t x(\tau) h(t-\tau) d\tau . \qquad \textbf{Eq. (3.5-1)}$$

The computer system now calculates the impulse response *h(t)* in the time domain. For the investigations about the pick-ups the transfer function $H(j\omega)$ is of interest. In the frequency domain the output signal $Y(j\omega)$ is defined as a multiplication of the input signal $X(j\omega)$ with the transfer function $H(j\omega)$. The signals in the frequency domain are linked by Eq. (3.5-2) and the transfer function can be computed by Eq.(3.5-3). [1]

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) . \qquad \textbf{Eq. (3.5-2)}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} . \qquad \textbf{Eq. (3.5-3)}$$

The impulse response and the transfer function of the system are linked with the Fourier Transform. This is also the way in which the computer calculates the transfer function of the system. The Fourier Transform can be expressed by the following equation. [1]

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = F\{h(t)\} . \qquad \textbf{Eq. (3.5-4)}$$

[1] [18] page 21-25

It is only necessary to compute the frequencies up to 20 kHz, in view of the range of audibility. The input signal *x(t)* (excitation signal) must be a function, which includes all frequencies from 0 Hz up to 20 kHz. This is the requirement that the denominator of Eq. (3.5-3) is not zero for any frequency. In reality, the QuickSig does computation with discrete-time signals and Fast Fourier Transform FFT (more about FFT [18]).

In theory it could be used any excitation signal that meets the requirements above. The excitation signal used by QuickSig in these measurements was an exponential function and it is represented in the following equation and graph.

$$x(t) = 2V \cdot e^{-\left(\frac{t}{T}\right)}. \qquad \textbf{Eq. (3.5-5)}$$

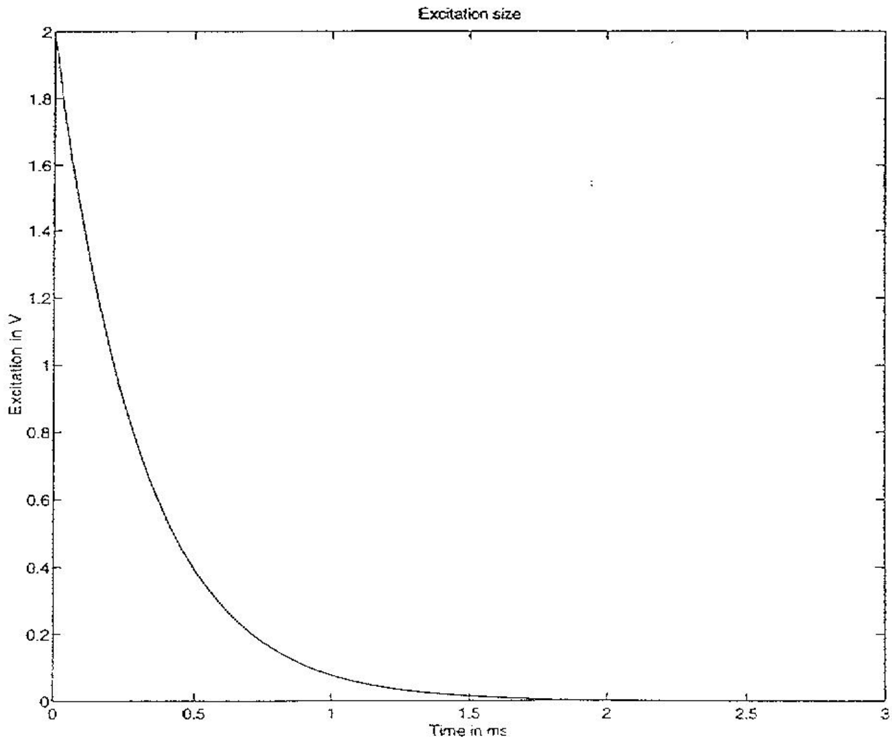


Fig. 3.13 Excitation signal used in measurements.

This excitation signal has a continuous spectrum and it can be calculated with the Fourier Transform. The following equation presents only the magnitude of the continuos spectrum.

$$\left|H_{exc}(j\omega)\right| = \frac{2V}{\sqrt{1+(\omega T)^2}}. \qquad \textbf{Eq. (3.5-6)}$$

The time constant T is about 0.4 ms and with this assumption the magnitude of the excitation signal at 20 kHz is approximately 40 mV. The reason why such an exponential excitation was used is that its spectrum has more emphasis and improved signal-to-noise ratio at low frequencies. This was found to give more reliable results.

For the measurements it has to be considered that the input resistance R_i of the amplifier, as well as the cable capacitance C_c were connected parallel to the pick-up. It has a big influence on the resonance frequency and on the absolute value of the impedance. The pick-up is driven by current source, i.e. the resistance R_{sh} is high (Fig. 3.14), so that the voltage U_p over the coil is proportional to the impedance. Therefore, the ratio of the pick-up voltage U_p and the input voltage U_i represent the impedance. The amplification factor k_a of the amplifier is considered during the analysis of the results with MATLAB. The QuickSig system can calculate with the parameter *h(t)* the transfer function of the system. The transfer function depends on the frequency properties of the impedance Z. Good results could be achieved with a shunt resistance R_{sh} of 3 $M\Omega$.

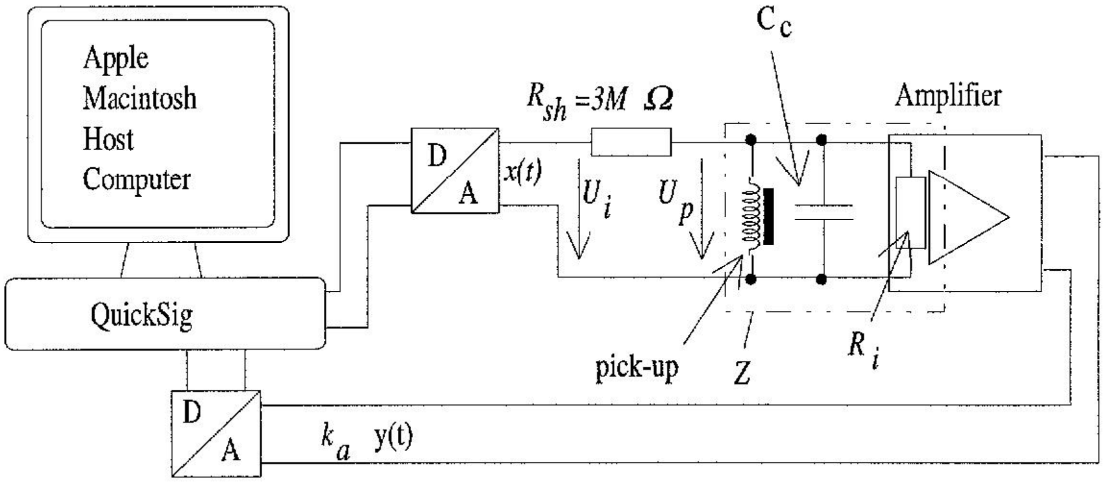


Fig. 3.14 Measuring arrangement for the impedance.

It has to be mentioned that the result of this measurement is not exactly the absolute value of the impedance Z. It is the ratio of the impedance Z with the impedance Z_e of the whole circuit (series connection of R_{sh} and Z). The absolute value of the impedance Z_e can be assumed to be constant, because of the high shunt resistant R_{sh}. This can be expressed with the following equation.

$$Z = \frac{U_p}{U_i} \cdot Z_e \rightarrow |Z| \approx \frac{|U_p|}{|U_i|} \cdot const. \qquad \textbf{Eq. (3.5-7)}$$

In Fig. 3.14 the cable capacitance C_c and the capacitance C of the pick-up (Fig. 3.2) can be summarised. The same can be done with the input resistance R_i and the loss resistance R_l (Fig. 3.2). Both are parallel connections. The results of these summaries are the total capacitance C_t and the total resistance R_t (table 3.9). The measurements with this measuring arrangement provided the resonance frequency f_{01} (table 3.9) and the total capacitance could be calculated with Eq. (3.3-2).

With the measured impedance curve and the computations by MATLAB the input resistance R_i was determined. For these computations Eq. (3.1-4) was used and the computed impedance curve was compared with the measured curve.

The determination of R_i gives in: $R_i = 824\ \mathrm{k\Omega}$.

With the loss resistances from table 3.7 and table 3.8 and the input resistance R_i it was possible to calculate the total resistance R_t of the pick-ups (parallel connection).

In the following table the results of the measurements are listed.

Pick-up	**Resonance frequency f_{0l}/kHz**	**Total capacitance C_t/pF**	**Total resistance R_t/kΩ**	**Max. impedance Z/kΩ**
Strato. Nr.1	8.27	134	537	456
Strato. Nr.2	8.66	125	570	490
Strato. Nr.3	8.27	137	554	467
Strato. Nr.4	8.40	132	565	480
Strato. Nr.5	8.40	131	565	481
Strato. Nr.6	8.66	123	559	484
Strato. Nr.7	8.53	130	574	487
Strato. Nr.8	8.48	129	552	475
Strato. Nr.9	8.01	150	553	459
Strato. Nr.10	8.57	128	562	481
Strato. Nr.11	8.74	122	566	489
Strato. Nr.12	8.27	133	543	463
Strato. Nr.13	8.53	129	546	460
Strato. Nr.14	8.66	125	562	484
Strato. Nr.15	8.91	131	537	454
Gretsch 1	9.86	68	463	429
Gretsch 2	15.86	49	505	464
voice proto 1	12.14	82	650	548
voice proto 2	9.65	79	654	590
voice proto 3	14.77	84	601	498
Fender Hum.	6.12	141	406	361
Sey. D. P-90	4.52	176	324	305
S. D. Hum. Jeff	7.75	199	238	218
Fender Lace 1	7.19	219	331	280
Fender Lace 2	2.89	386	498	369

Table 3.9 Measurement of the impedance with QuickSig.

Table 3.9 shows the differences between the measured resonance frequencies f_{0l} and the resonance frequencies f_0 from section 3.4.1-2. The reason for these differences can be attributed to the relatively high cable capacitance. The cable capacitance C_c can be calculated as the difference between the total capacitance C_t and the capacitance C of the pick-up (C_c is approximately 50 pF).

The measurements with QuickSig confirm the trend of the pick-ups to have different resonance frequencies. It is shown that pick-up Nr. 9 tends to have a lower resonance frequency than the other *Stratocaster* pick-ups. It also confirms the high resonance frequency of the *Gretsch 2* pick-up as well the low resonance frequency of the *Fender Lace 2* pick-up.

The effect of the input resistance R_i can be recognised at the lower total values of the impedance Z. The impedance Z is the parallel connection of the pick-up, the capacitance C_c, and the input resistance R_i.

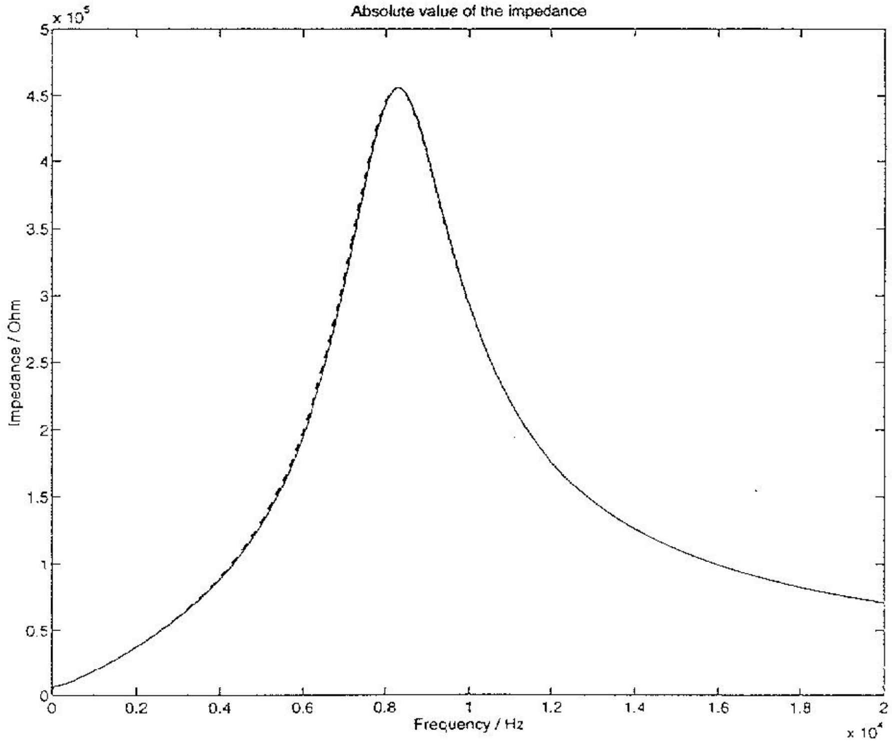


Fig. 3.15 Calculated and measured impedance of the *Stratocaster* pick-up Nr. 1.

The analysis of the measurements with MATLAB shows the agreement of the measured curve (dashed line) and the calculated curve (Eq. (3.1-4)). The good results for the *Stratocaster* pick-ups are noticeable. There is almost no difference between the calculated and the measured curve.

These results confirm that the equivalent circuit diagram from section 3.1 fits for the *Stratocaster* pick-ups. For the other pick-ups the results are acceptable. All measured curves of the pick-ups can be seen in Appendix C. The deviation of the calculated curve from the measured curve is small in all cases. The biggest deviation can be noticed in the measurements with the *De Armond/Dynasonic 1957 Gretsch 1/2*. This deviation is still acceptable and it can be noticed, that the equivalent circuit diagram from section 3.1 is valid.

All MATLAB programs for the diagrams in this section can be seen in Appendix B. Every MATLAB program includes comments, so that it is possible to follow the commands.

The QuickSig program is also integrated into Appendix B but without comments. Refer to professor Matti Karjalainen, Helsinki University of Technology (Acoustics Laboratory), who has written the program.

3.7.3 Calculation of the theoretical transfer function U_1/U_0

For the electric behaviour of the pick-ups it is important to reflect the transfer function U_1/U_0. This is a sign in which manner frequencies are amplified or attenuated. Every pick-up has its own distinctive transfer function and it depends strongly on the impedance Z. The next figure shows the equivalent circuit diagram with cable capacitance C_c and input resistance R_i of the amplifier.

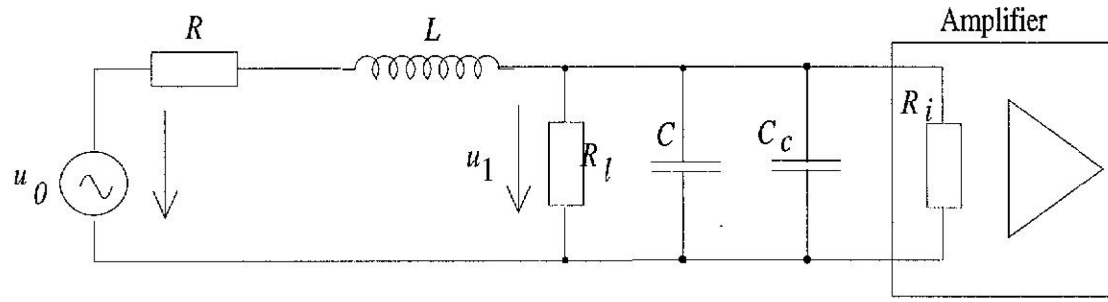


Fig. 3.16 Pick-up with cable capacitance C_c and input resistance R_i.

It was shown on page 57 that the capacitances C and C_c and as well the resistances R_l and R_i can be combined. The capacitance C_c and the resistance R_i are not considered for the derivation of the transfer function. Therefore, the following equation is valid.

$$\frac{U_0}{R + j\omega L + \dfrac{1}{\dfrac{1}{R_l} + j\omega C}} = \frac{U_1}{\dfrac{1}{\dfrac{1}{R_l} + j\omega C}} \qquad \textbf{Eq. (3.6-1)}$$

After several mathematical transformations it can be written as:

$$\frac{\underline{U_1}}{\underline{U_0}} = \frac{\frac{1}{R + j\omega L}}{\frac{1}{R + j\omega L} + \frac{1}{R_l} + j\omega L} \qquad \textbf{Eq. (3.6-2)}$$

If Eq. (3.6-2) is compared with Eq. (3.1-1) it can be recognised that Eq. (3.6-2) includes the impedance Z of the pick-up. For further investigations the magnitude of the transfer function is of interest. In view of this, Eq. (3.6-2) can be expressed as:

$$\frac{U_1}{U_0} = \frac{|Z|}{\sqrt{R^2 + (\omega L)^2}} \qquad \textbf{Eq. (3.6-3)}$$

Eq. (3.6-3) shows the dependence of the transfer function from the absolute value of the impedance Z. If the cable capacitance and the input resistance are considered, then the resonance frequency and the absolute value of the transfer function would be influenced (see section 3.8).

For the theoretical calculations of the transfer functions with Eq. (3.6-3) the load of Table 3.9 was assumed. The transfer function is usually represented in *db* and in a logarithmic frequency axis. In Fig. 3.17 the theoretical transfer functions of five different pick ups are shown. These are:

1) *SSL-1 Vintage pick-up for Stratocaster guitar Nr.1 (Seymor Duncan)*
2) *De Armond/Dynasonic 1957 Gretsch 2*
3) *Voice pick-up proto 3*
4) Fender-Lace Sensor 2
5) *Fender Humbucker*

Fig. 3.17 shows the typical peaks of pick-ups at different resonance frequencies. The similar resonance frequencies, compared with the impedance measurements (Table 3.9), are outstanding.

The theoretical curves of all pick-ups can be seen in the Appendix D.

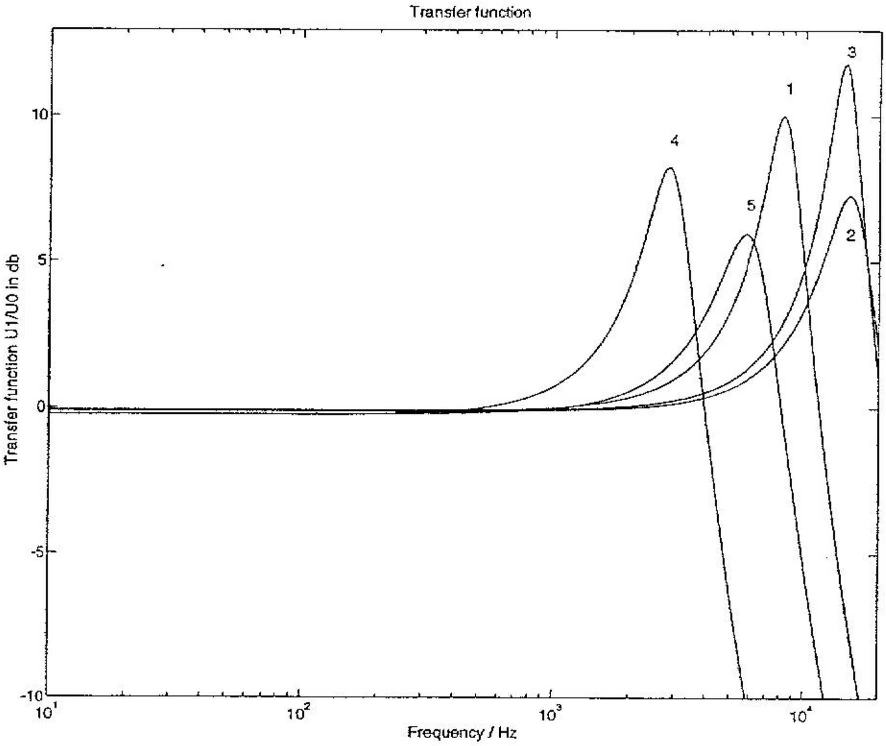


Fig. 3.17 Theoretical transfer function of five different pick-ups.

3.7.4 Measurement with excitation coil

The following figure shows the measuring arrangement with an excitation coil.

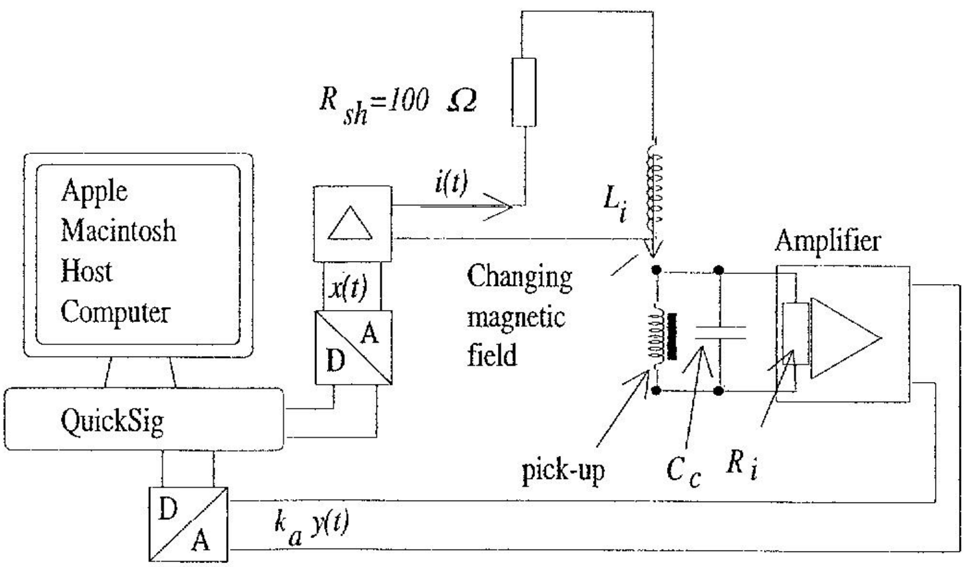


Fig. 3.18 Measuring arrangement with excitation coil.

The inductance L_i (L_i=1.87 µH) is driven by a current source and it generates a magnetic field according to the current $i(t)$ ($i(t)=x(t)/R_{sh}$). If the generated magnetic field intersects the pick-up a voltage u_0 (Eq. (2.5-1)) is induced into the coil.
The amplitude of the induced voltage u_0 depends on the differential $d\Phi/dt$ and therefore it depends also on the frequency f of the input signal x(t). This is shown in the following equation.

$$\frac{d\Phi}{dt} = \frac{d\hat{\Phi}\sin(2\pi ft)}{dt} = \hat{\Phi}2\pi f\cos(2\pi ft) \qquad \textbf{Eq. (3.6-4)}$$

$\hat{\Phi}$ amplitude of the magnetic flux.

At low frequencies the induced voltage u_0 is approximately 0 V and therefore the output voltage u_1 is 0 V. QuickSig calculates the transfer function of the system which includes the generation of the voltage u_0 and the transfer function U_1/U_0 of the pick-up. This measurement arrangement provides the possibility to measure the resonance frequency f_{02}. The measured results of f_{02} (Table 3.10) are similar in comparison with the resonance frequencies f_{01} from section 3.7.2.

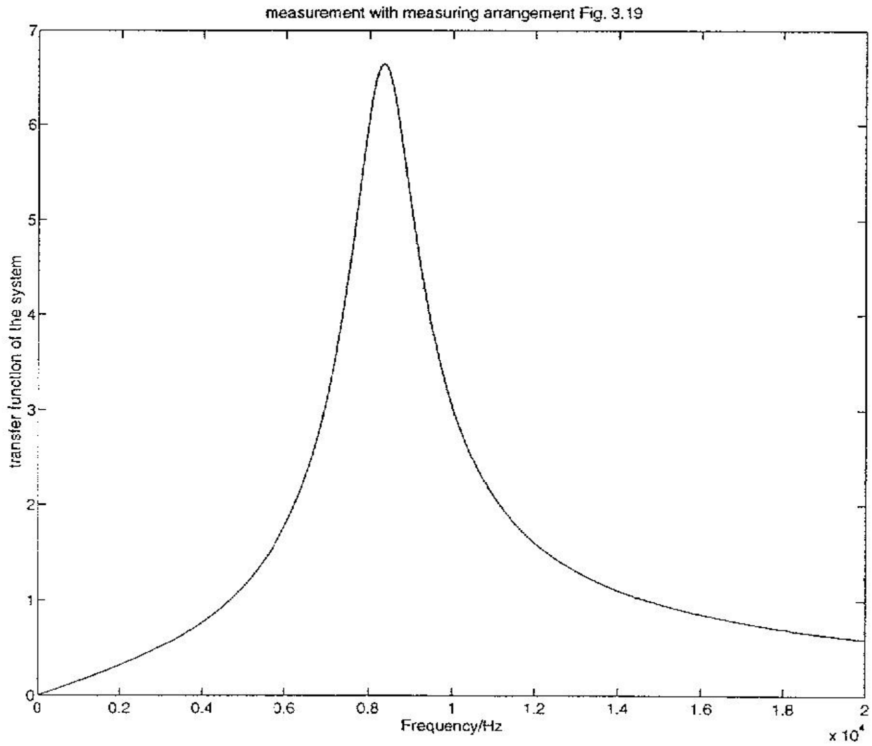


Fig. 3.19 Measured curve of *Stratocaster* pick-up Nr. 1 with measuring arrangement of Fig 3.18

Fig. 3.19 shows the distinct resonance frequency of *Stratocaster* pick-up Nr. 1 and it confirms that the output voltage at low frequencies is approximately 0 V.

Pick-up	Resonance frequency f_{02} /kHz	Pick-up	Resonance frequency f_{02}/kHz	Pick-up	Resonance frequency f_{02}/kHz
Strat. Nr1	8.35	*Strat. Nr10*	8.44	*Voice 1*	13.09
Strat. Nr2	8.74	*Strat. Nr11*	8.61	*Voice 2*	10.38
Strat. Nr3	8.27	*Strat. Nr12*	8.10	*Voice 3*	15.33
Strat. Nr4	8.44	*Strat. Nr13*	8.57	*FenderHum*	6.03
Strat. Nr5	8.40	*Strat. Nr14*	8.61	*S.D. P-90*	4.48
Strat. Nr6	8.53	*Strat. Nr15*	9.04	*S.D.Hum*	7.84
Strat. Nr7	8.48			*Lace Sen.1*	7.15
Strat. Nr8	8.44	*Gretsch 1*	10.08	*Lace Sen.2*	2.89
Strat. Nr9	8.01	*Gretsch 2*	15.37		

Table 3.10 Resonance frequencies with measuring arrangement 2.

If table 3.10 is compared with table 3.9. it is obvious that the resonance frequencies are similar. Both measurements had almost the same cable capacitance and the results show the possibility of measuring arrangement 2 to determine the resonance frequency.

3.8 Influence of the connected cables and amplifiers

If an electric guitar is played then it is usually connected to an amplifier with guitar cables. These guitar cables have usually capacitances between 500 pF and 3 nF and the input resistance of the amplifier is around $1\mathrm{M\Omega}$. It was previously shown that parallel connected components have a big effect on the impedance and the transfer function. Most guitars have a volume control and a tone control which are fixed directly to the guitar (Fig. 3.22). The volume control consists of a parallel connected potentiometer. The tone control is a series combination of a capacitance and a potentiometer which is connected parallel to the pick-up. For the sound of a guitar it is important to know the electric data of the used cable, volume control, tone control and amplifier. With this knowledge it is possible to know the electric behaviour of the pick-up which creates the sound. In this section the influence of different parallel connected components are simulated.

3.8.1 Influence of different cables

In the following figure a very high input resistance is assumed (infinite) and the cable capacitance is varied between 0 pF and 3 nF (0 pF, 150 pF, 470 pF, 1 nF, 3 nF). The simulations were done with the *Stratocaster* pick-up Nr.1.

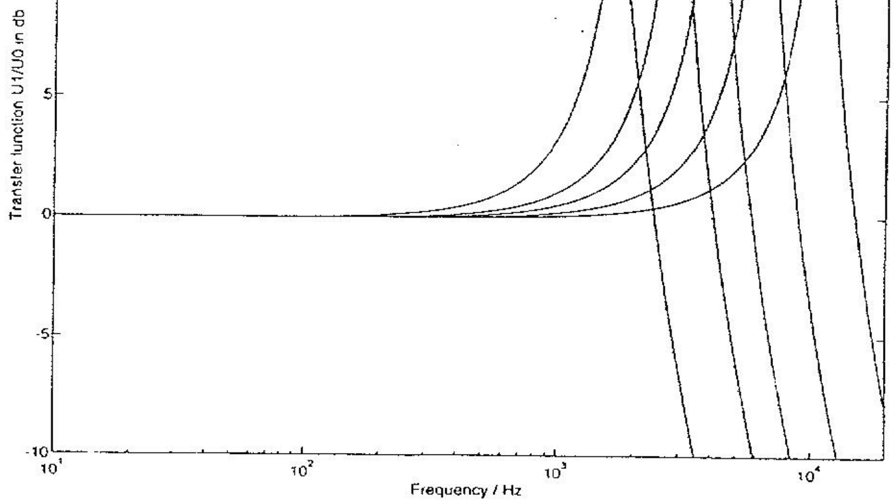


Fig. 3.20 Influence of different cables.

The simulation shows the effect of the cable capacitance on the resonance frequency. If the cable capacitance increases then the resonance frequency decreases. This shows the possibility to get a different sound with different cables. Another way is to connect different capacitances to a switch, and the guitarist has the choice between different parallel connected capacitances. It is remarkable that the peak height of the resonance frequency is also influenced by the capacitance.

3.8.2 Influence of different amplifier input resistances

In the following figure a cable capacitance of 0 pF is assumed and the input resistance is varied between 10 MΩ and 10 kΩ (10 MΩ, 1 MΩ, 300 kΩ, 100 kΩ, 10 kΩ).

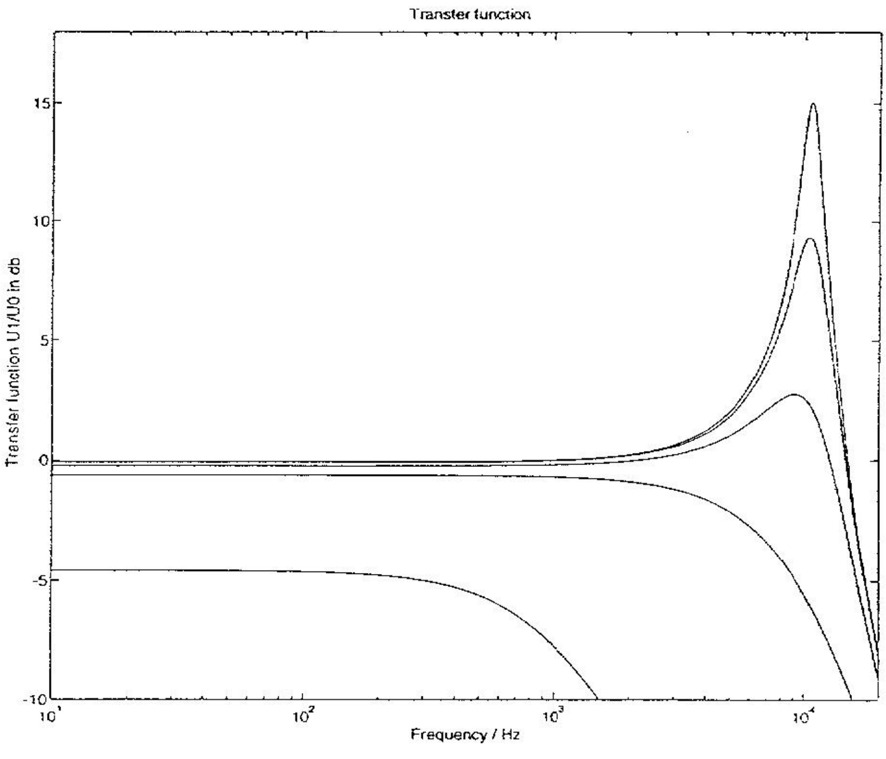


Fig. 3.21 Influence of the input resistance.

The figure above shows the attenuation of lower input resistances. The curves with 10 kΩ and 100 kΩ have no distinct peak and the quality of sound will be reduced. It is better to avoid lower input resistances than 300 kΩ to get a distinct peak and an acceptable sound. With very low input resistances the induced voltage u_0 is already attenuated at low frequencies. As long as the transfer function has a resonant peak, the resonance frequency is weakly influenced by the input resistance.

3.8.3 Tone and volume control

The following figure shows the typical connections of tone control and volume control.

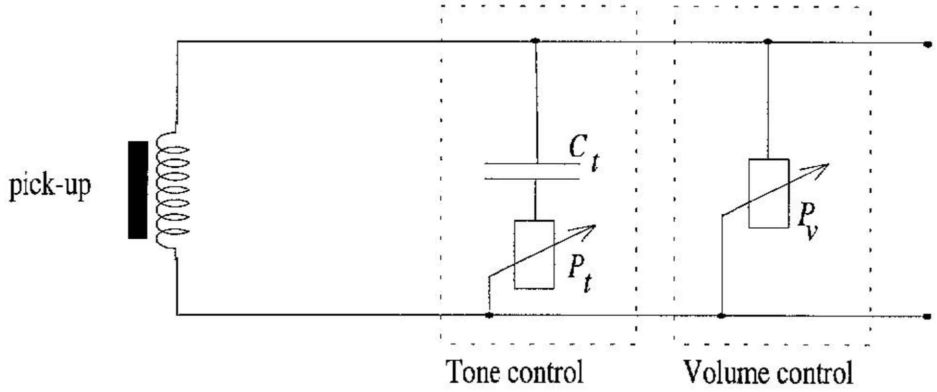


Fig. 3.22 Connections of tone control and volume control.

A volume control consists of a potentiometer P_v. Its valuc is usually in a range from 100 kΩ to 1 MΩ and its effects are shown in Fig. 3.21. A smaller value of P_v gives a lower output voltage and it influences the sound of the guitar in view of the damped resonant peak.

The tone control is a series connection of a capacitance C_t (ca. 1nF) and a potentiometer P_t (250 kΩ - 1 MΩ). If the potentiometer is turned down to smaller values the resonance frequency rises and the damping of the resonant peak is higher.

A few measurements were done to show the interaction of volume and tone control (With maximum values of the potentiometers) and it shows the electric behaviour of guitars in use. The measurements were done with measuring arrangement 2 (Fig. 3.18) and the transfer function of the system was measured. For the calculation of the transfer function it is necessary to take the voltage over the coil L_i as the input signal $x(t)$. This was done due to the proportionality to the induced voltage u_0. Both voltages depend on the time derivative of the magnetic field. The results of the measurements can be seen in Appendix E.

The results show that the pick-ups do not have any distinctive resonance frequency without any load. With a load of 820 pF the pick-ups get a resonance frequency and they have a small peak.

3.9 The magnetic field of pick-ups

Every pick-up generates non-linear distortion because of the non-linear correlation between magnetic flux and string distance to the top of the magnetic pole piece. The shape of the magnetic flux is shown in the following figure.

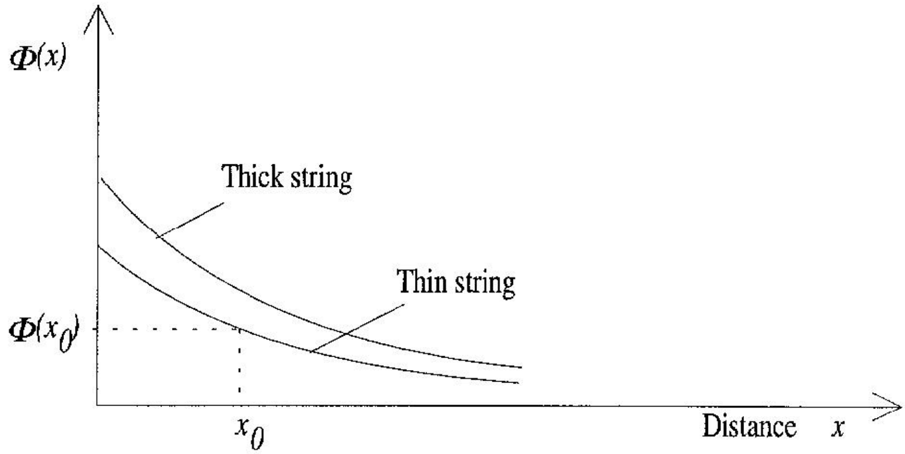


Fig. 3.23 Magnetic flux in dependence on the distance between string and magnetic pole piece.

The physical condition of the magnetic flux depends on the distance x of the string to the magnetic pole-piece, the thickness of the string and of its permeability. If the string approaches to the magnetic pole-piece then the change of the magnetic flux rises the steeper the closer the string is from the pole-piece (Fig. 3.23).

In the neutral position the string has the distance x_0 and the magnetic flux is $\Phi(x_0)$. If x_0 is small then a small vibration amplitude would produce a relatively high change of the magnetic flux. On the other hand a big neutral distance x_0 with a big vibration amplitude produces a relatively small change of the magnetic flux. This correlation is noticeable at the output voltage of the pick-up.

The curve of Fig. 3.23 can be expressed with the Taylor series at the distance x_0. [1]

$$\phi(x)=\phi(x_0)+\frac{\phi'(x_0)}{1!}\cdot(x-x_0)+\frac{\phi''(x_0)}{2!}(x-x_0)^2+\frac{\phi'''(x_0)}{3!}(x-x_0)^3\ldots \quad \textbf{Eq. (3.7-1)}$$

The induced voltage is proportional to the differential $d\Phi/dt$ (Eq. (2.5-1)). Under the assumption of a pure sinusoidal string vibration, it is possible to calculate the differential with the following equation.

$$\frac{d\phi}{dt}=\phi'(x_0)\frac{d\sin(\omega t)}{dt}+\frac{1}{2}\phi''(x_0)\left(\frac{d\sin^2(\omega t)}{dt}-2x_0\frac{d\sin(\omega t)}{dt}\right)+\ldots \qquad \textbf{Eq. (3.7-2)}$$

[1] [9] page 477

The induced voltage is not exactly proportional to the velocity *dx/dt* of the string vibration. In addition to the sinusoidal signal there will be signal components due to the higher powers. Trigonometric transformations and the differentiation of them would show that the additional terms are natural multiples of the fundamental. These are the non-linear distortions. A higher vibration amplitude will cause a higher non-linear distortion. Under the assumption of an infinitely small vibration amplitude the induced voltage will be proportional to the velocity of the string vibration. The following picture shows an output voltage of a pick-up with a sinusoidal string vibration (200 Hz).

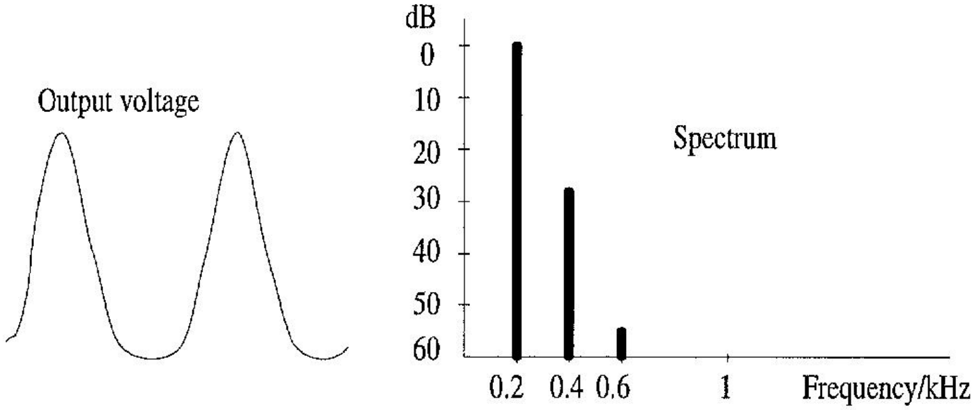


Fig. 3.24 [1] Output voltage and spectrum of a pick up with a sinusoidal string vibration (f=200 Hz).

The generated frequencies are harmonics of the fundamental and in consideration of this the sound will be more mellow. Usually the string vibration is not only the fundamental (section 2.5), it also vibrates with the harmonics. These harmonics generate sum frequencies and difference frequencies which are also harmonics of the fundamental.

When two strings are excited the sum and difference frequencies may also be non-harmonic frequencies . This effect is called intermodulation and the sound will be more impure. The intermodulation effect is smaller when the strings are more away from the magnetic pole-piece and a stronger magnetic field causes more intermodulation. [2]

In one measurement it was tried to pluck the string as consistently as possible and the distance to the pole-pieces was altered. An oscilloscope was used to measure the output voltage of the pick-up. This measurement was quite inaccurate but it has shown that the sensitivity of a pick-up is close to being proportional to the inverse of the distance.

Furthermore it has to be mentioned the unequal sensitivity of the pick-up to different string vibrations. A pick-up is more sensitive to vibrations perpendicular to the guitar than to vibrations in a parallel plane.

[1] [1] page 89
[2] [1] page 90 and [11] page 46-48

3.9.1 Measurements of the magnetic field

The measurements were done with the *Stratocatser* pick-up Nr.1. The magnetic field was measured from the top of the magnetic pole-pieces up to a distance of 1 cm. These measurements are rough due to problems to adjust correctly the distances. The used measuring device was a "*Bell 610 Gaussmeter*". It measured the magnetic field density in [Gauss]. The measuring unit was converted in [Tesla] (1 G = 10^{-4} T).

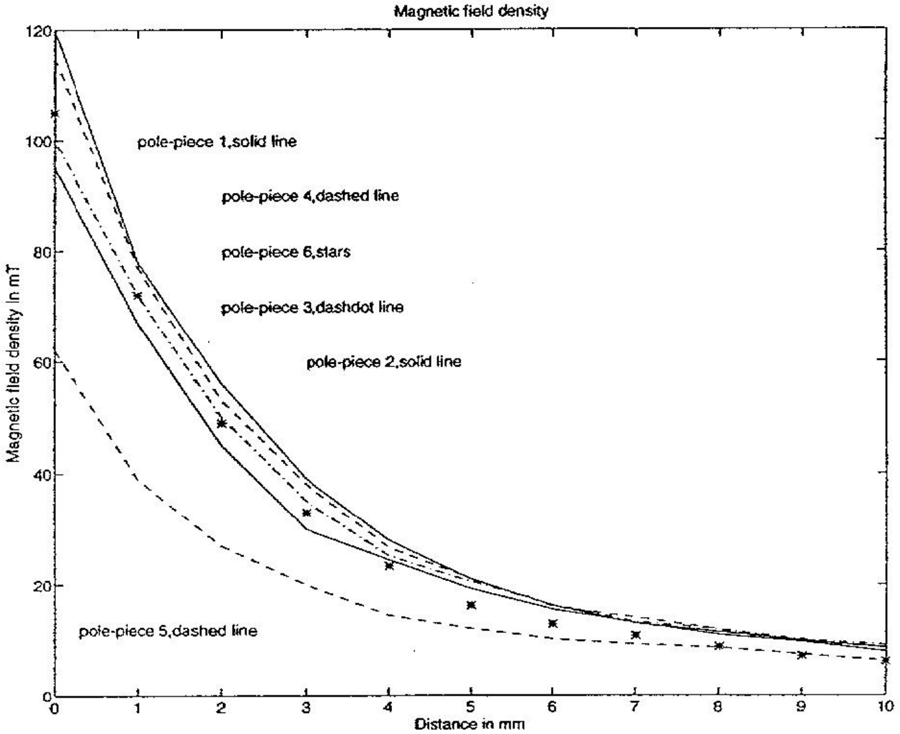


Fig. 3.25 Magnetic field density of *Stratocaster* pick-up Nr.1 as a function of the distance from the pole-piece.

The figure above shows the expected shape of the magnetic field. The magnetic field of pole-piece Nr. 5 is remarkable in view of its obviously weaker magnetic field. The strings are usually adjusted between a height of 3 mm and 5 mm but it has to be noticed that already the pole-pieces are adjusted to different heights (This is desireable to compensate the different magnetic sensitivities of the different strings).

3.9.2 Measurements of the magnetic field with *Stratocaster* pick-ups

For the comparison of the different magnetic field densities, it was measured only on the top of the magnetic pole-pieces.

Pick-up	Magnet Nr1 B/mT	Magnet Nr2 B/mT	Magnet Nr3 B/mT	Magnet Nr4 B/mT	Magnet Nr5 B/mT	Magnet Nr6 B/mT
Strat.1	115	94	97	104	60	108
Strat.2	125	120	116	104	100	120
Strat.3	132	122	117	124	101	115
Strat.4	133	111	110	127	91	116
Strat.5	127	115	116	122	100	114
Strat.6	140	118	112	120	105	119
Strat.7	120	110	122	129	102	120
Strat.8	125	114	127	125	94	120
Strat.9	112	101	97	104	94	111
Strat.10	119	105	109	108	93	119
Strat.11	120	111	115	124	102	115
Strat.12	119	119	111	124	63	123
Strat.13	129	106	114	130	105	119
Strat.14	110	101	111	125	102	119
Strat.15	129	113	104	115	91	109

Table 3.11 Magnetic field density on the top of the *Stratocaster* pole-pieces.

The Magnetic field density of the different pick-ups has similar values for every pole-piece. Only the big differences of the pole-piece 5 are outstanding. The mean value for the magnetic field strength of all pole-pieces is about 115 mT. It has to be mentioned that the pick-up alters its magnetic field density as it ages.

Some measurement results:

Kind of value	Magnet Nr1	Magnet Nr2	Magnet Nr3	Magnet Nr4	Magnet Nr5	Magnet Nr6
Max.	140 mT	122 mT	127 mT	130 mT	105 mT	123 mT
Min	110 mT	94 mT	97 mT	104 mT	60 mT	108 mT
Mean	124 mT	111 mT	112 mT	119 mT	93.5 mT	116 mT
Standard deviation	8.3 mT	8.0 mT	8.1 mT	9.5 mT	13.9 mT	4.4 mT

Table 3.12 Maximum, minimum, mean value, and standard deviation

3.9.3 Measurements of the magnetic field with different pick-ups

Pick-up	Magnet Nr1 B/mT	Magnet Nr2 B/mT	Magnet Nr3 B/mT	Magnet Nr4 B/mT	Magnet Nr5 B/mT	Magnet Nr6 B/mT
Gretsch 1	131	110	101	105	101	124
Gretsch 2	121	105	99	102	103	110
Voice 1	104	83	88	83	94	102
Voice 2	90	94	94	87	78	89
Voice 3	88	101	88	81	88	96
S.D. P-90	32	27	26	27	29	34

Table 3.13 Magnetic field density of the single pick-ups.

The *Fender Lace Sensor* pick-ups do not have 6 magnetic pole-pieces and therefore the magnetic field density measurement was done on the top cover of the pick-ups.

Fender Lace Sensor 1 : B = 29 mT
Fender Lace Sensor 2 : B = 32 mT

The *Seymor Duncan P-90* and the *Fender Lace Sensor* pick-ups have a quite weak magnetic field in comparison to the other pick-ups. It is obvious that the *Stratocaster* pick-ups have the strongest magnetic field of all pick-ups.

3.9.4 Measurements of the magnetic field with humbucker pick-ups

Humbucker pick-up		Magnet Nr1 B/mT	Magnet Nr2 B/mT	Magnet Nr3 B/mT	Magnet Nr4 B/mT	Magnet Nr5 B/mT	Magnet Nr6 B/mT
Fender humbucker	row 1	42	35	38	24	25	30
	row 2	30	27	33	44	35	39
Seymor D. humbucker	row 1	45	42	40	38	37	38
	row 2	37	33	34	36	34	36

Table 3.14 Magnetic field density of the humbucker pick-ups.

In table 3.14 the quite weak magnetic field of humbucker pick-ups can be noticed. The humbucker pick-ups do not have necessarily a lower output voltage in comparison to pick-ups with stronger magnetic fields. This is understandable in view of the series connection of two single coil pick-ups (section 2.6.3.4). The comparison of all magnetic field measurements shows that the Stratocaster pick-ups have the strongest magnetic field. It has to be mentioned that the strength of the magnetic field is not a sign for more quality. In this section it was already shown that a stronger magnetic field causes more distortion and intermodulation.

The quality of a pick-up depends on the interaction of all components and details which were mentioned during the whole thesis. Furthermore it depends on the desired sound of every guitarist and it is not possible to develop one pick-up that fulfils all wishes of different sounds.

4. Conclusion

This work has introduced a description of the electric guitar and elucidated the electric behaviour of pick-ups. The thesis provides theoretical background on details which determine the sound generation and the possibilities to influence the sound in a desired way. With different measurements an equivalent circuit diagram was determined. The derived equivalent circuit diagram of the pick-up was found to be suitable for a frequency range from 0 Hz to 20 kHz. For all measured pick-ups the values of the components of the equivalent circuit diagram were determined. Measurements with the QuickSig system were a new method to confirm the accuracy of the equivalent circuit diagram. Furthermore, some investigations and measurements on the magnetic field have been included.

The analysis of the measurements has shown the distinctive differences in the electric behaviour of pick-ups. The Stratocaster pick-ups, which were of the same type, were also confirmed to have uniform electric behaviour. Measurements of pick-ups with a volume and tone control have shown the differences in the behaviour of connected and disconnected pick-ups.

The results have shown the ability of the QuickSig system to measure different electric properties of the pick-ups. Further investigations on pick-ups could be based on the results of this thesis. The measurements could be repeated with other pick-ups, especially the differences of bass pick-ups would be worth to study. Such parts of the thesis which were only studied in theory could be confirmed by measurements. For instance nonlinear distortion and intermodulation due to the nonlinear magnetic field. The next step could be studies on the whole "electric guitar system", including the strings, the pick-up, and the body.

List of abbreviations and symbols

AC	Alternating current
A/D	Analog/Digital
AI	Artificial Intelligence
AWG	American Wire Gauge
CLOS	Common Lisp Object System
D/A	Digital/Analog
DC	Direct current
DSP	Digital Signal Processing
i.e.	Id est
kg	Kilogram
m	Meter
N	Newton
OOP	Object Oriented Programming
A_i	Amplitudes of string vibration
A_p	Area of the capacitor
a_i	Vibration position
A_w	Wire cross section
B	Magnetic flux density
C	Capacitance of the pick-up
c	Speed of travelling waves
C_c	Cable capacitance
C_E	Capacitance to the case
C_G	Capacitance to ground
C_n	Coefficient for calculation of the amplitudes
C_t	Total capacitance
C_W	Winding capacitance
d	Diameter of the string
d_p	Distance between the plates
E	Elastic modulus
f_n	Frequency in dependence on n
f_0	Self-resonant frequency of the pick-up
h	displacement at the excited point
$h(j\omega)$	Transfer function
$h(t)$	Impulse response
K	Constant
k	Tractive power
k_a	Amplification factor
L	Inductance
L_i	Inductance of the magnetic field circuit
l	String length
l_c	Element of a coil

l_w	Length of wire
m	Mass
N	Turns of wire in the coil
n	Number of the harmonic
Q_c	Quantity of electricity
R	Resistance
R_i	Input resistance
R_l	Loss resistance
R_{sh}	Shunt resistance
R_t	Total resistance
s	Standard deviation
T_1	Time constant 1
T_2	Time constant 2
t	Time
U_c	Voltage at the capacitance
$\|U_i\|$	Absolute input voltage
$\|U_p\|$	Absolute voltage at the pick-up
U_0	Effective value of the induced voltage
U_1	Effective value of the output voltage
u_g	Generated voltage
ul	Induced voltage of a dynamic microphone
u_0	Induced voltage of a pick-up
v	velocity
x	Position
x_0	Neutral position
$x(j\omega)$	Input signal in the frequency domain
$x(t)$	Input signal in the time domain
$\overline{x}$	Mean value
$y(j\omega)$	Output signal in the frequency domain
$y(t)$	Output signal in the time domain
Z	Impedance
$\|Z\|$	Absolute impedance of the pick-up
$\|Z_e\|$	Absolute impedance of the whole circuit
$ü$	Transmission factor
β_i	Damping rate
δ_w	Resistivity
ε_0	Relative permittivity
ε_r	Absolute permittivity
Φ	Magnetic flux
φ_m	Phase of motion

φ_p	Phase of the pick-up
λ	Wave length
ρ	Density of the string material
σ_z	Tension on the string
τ_{eff}	Effective string length
ω	Radian frequency
ω_0	Self-resonant radian frequency of the pick-up
ξ	Displacement

MATLAB program for the theoretical calculation of the impedance and the transfer of data from QuickSig to MATLAB

% Absolute impedance

% Calculated impedance

```
f=0:10:20000;                                   % matrix from 0 to 20000, step 10
»L=2.77;                                        % value of L
»R=6910;                                        % value of R
»C=131e-12;                                     % value of C
»Rv=0.537e6;                                    % value of Rv
»o=1+2*pi*f*(L/R)*i;                            % calculation of Z with equation 3.3
»w=2*pi*f.*sqrt(L*C);
»w=w.^2;
»u=1+(R/Rv)-w+2*pi*f*(L/Rv+R*C)*i;
»Z=R*(o./u);
»betr=abs(Z);                                   % absolute value of Z
»[impmax,fres]=max(betr)                        % maximum of magnitude
»plot(f,betr);                                  % plot betr as a function of f
»xlabel('Frequency / Hz');                      % x axis label
»ylabel('Impedance / Ohm');                     % y axis label
»title('Absolute value of the impedance');      % title
axis([0,20000,0,500000]);                       % determination of axes
hold on;                                        % hold plot
```

% Transfer of data from QuickSig to MATLAB
% Measured impedance

```
f=sig_from_QS;                                  % signal from QS
»x=1:20000/465:20000;                           % matrix from 1 to 20000, step 20000/4
»f=456000/4.65*f;                               % determination of the impedance
»plot(x,f,'--');                                % plot f as a function of x, dashed line
```

Program for the calculation of resistance and reactance

```
% Calculation of resistance and reactance

% 1) Resistance

f=0:10:20000;                          % matrix from 0 to 20000, step 10
»L=3.46;                               % value of L
»R=9620;                               % value of R
»C=30.7e-12;                           % value of C
»Rv=3.168e6;                           % value of Rv
»o=1+2*pi*f*(L/R)*i;                   % calculation of Z
»w=2*pi*f.*sqrt(L*C);
»w=w.^2;
»u=1+(R/Rv)-w+2*pi*f*(L/Rv+R*C)*i;
»Z=R*(o./u);r=real(Z);                 % real part of Z
»plot(f,r);                            % plot real part
hold on                                % hold plot

% 2) Reactance

f=0:10:20000;                          % matrix from 0 to 20000, step 10
»L=3.46;                               % value of L
»R=9620;                               % value of R
»C=30.7e-12;                           % value of C
»Rv=3.168e6;                           % value of Rv
»o=1+2*pi*f*(L/R)*i;                   % calculation of Z
»w=2*pi*f.*sqrt(L*C);
»w=w.^2;
»u=1+(R/Rv)-w+2*pi*f*(L/Rv+R*C)*i;
»Z=R*(o./u);imz=imag(Z);               % imaginary part of Z
»plot(f,imz);                          % plot imaginary part of Z
hold on                                % hold plot
```

Program for the calculation of the phase

```
                    % Calculation of the phase

L=3.46;                                    % value of L
»R=9620;                                   % value of R
»C=30.7e-12;                               % value of C
»Rv=3.168e6;                               % value of Rv
f0=1/(2*pi*sqrt(L*C));                     % calculation of f0
f1=0:10:f0+20;                             % matrix from 0 to f0+20
»phaseup=atan(2*pi*f1*L/R)*360/(2*pi);     % calculation of the phase
»up=2*pi*f1*(L/Rv+R*C);                    % with equation 3.5
m=2*pi*f1.*sqrt(L*C);
»m=m.^2;
»down=1+R/Rv-m;
phasedown=atan(up./down)*360/(2*pi);
»phase=phaseup-phasedown;
»plot(f1,phase);                           % phase plot part one
hold on                                    % hold plot
f1=f0+40:10:20000;                         % matrix from f0+40 to 20000
»phaseup=atan(2*pi*f1*L/R)*360/(2*pi);     % calculation of the phase
»up=2*pi*f1*(L/Rv+R*C);                    % with equation 3.5
m=2*pi*f1.*sqrt(L*C);
»m=m.^2;
»down=1+R/Rv-m;
phasedown=atan(up./down)*360/(2*pi);
»phase=phaseup-phasedown-180;              % consideration of -180 degree
»plot(f1,phase);                           % phase plot second part
```

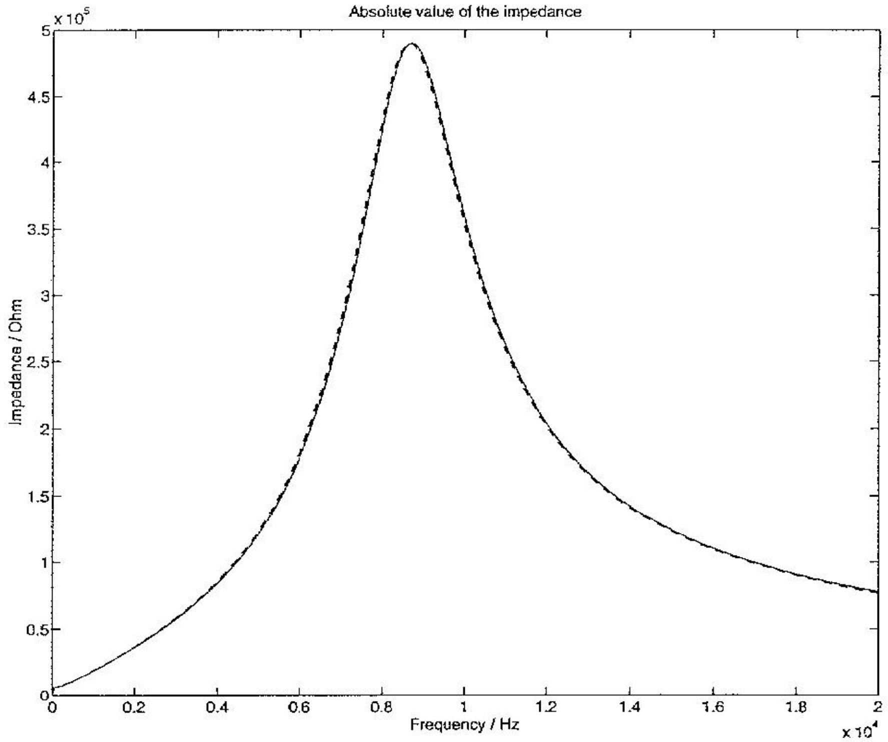
Program for the transfer function

% Calculation of the transfer function in db

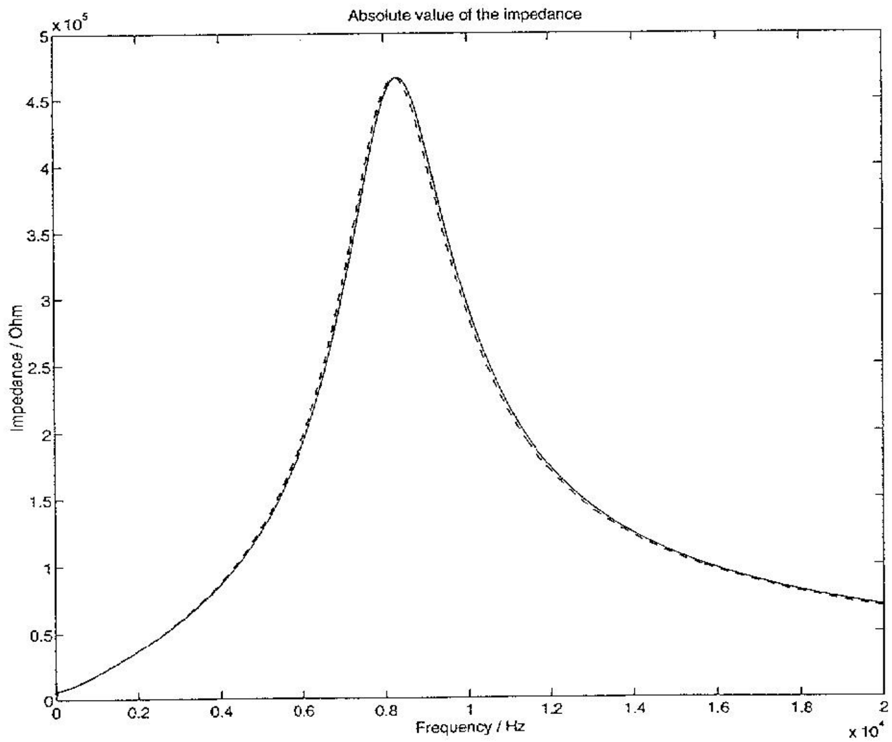
```
f=0:10:20000;                         % matrix from 0 to 20000, step 10
»L=2.77;                              % value of L
»R=6910;                              % value of R
»C=131e-12;                           % value of C
»Rv=0.537e6;                          % value of Rv
»o=1+2*pi*f*(L/R)*i;                  % calculation of transfer function
»w=2*pi*f.*sqrt(L*C);
»w=w.^2;
»u=1+(R/Rv)-w+2*pi*f*(L/Rv+R*C)*i;
»Z=R*(o./u);
»betr=abs(Z);
»tra=R+2*pi*f*L*i;
»tra=abs(tra);
»betr=betr./tra;
»db=log10(betr)*20;                   % calculation in db
»plot(f,betr);                        % plot
»[impmax,fres]=max(betr)              % maximum value in db
»xlabel('Frequency / Hz');            % x axis label
»ylabel('Transfer function U1/U0');   % y axis label
»title('Transfer function');          % title
»axis([0,20000,0,3.5]);               % determination of axis
»hold on;                             % hold plot
```

Diagrams of the measurements

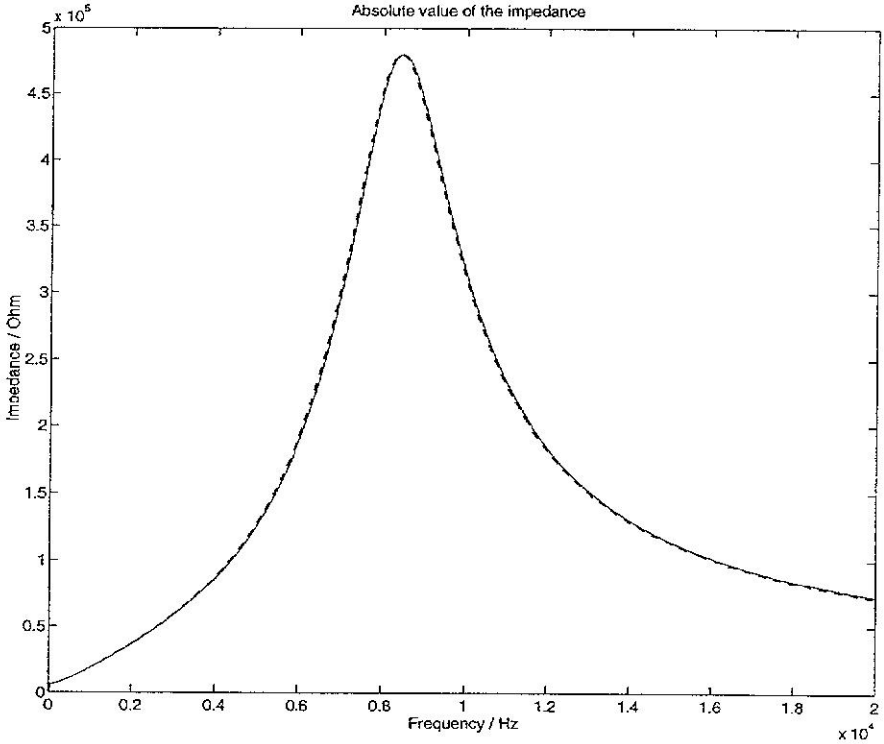
Dashed line is the measured curve and solid line is the calculated curve



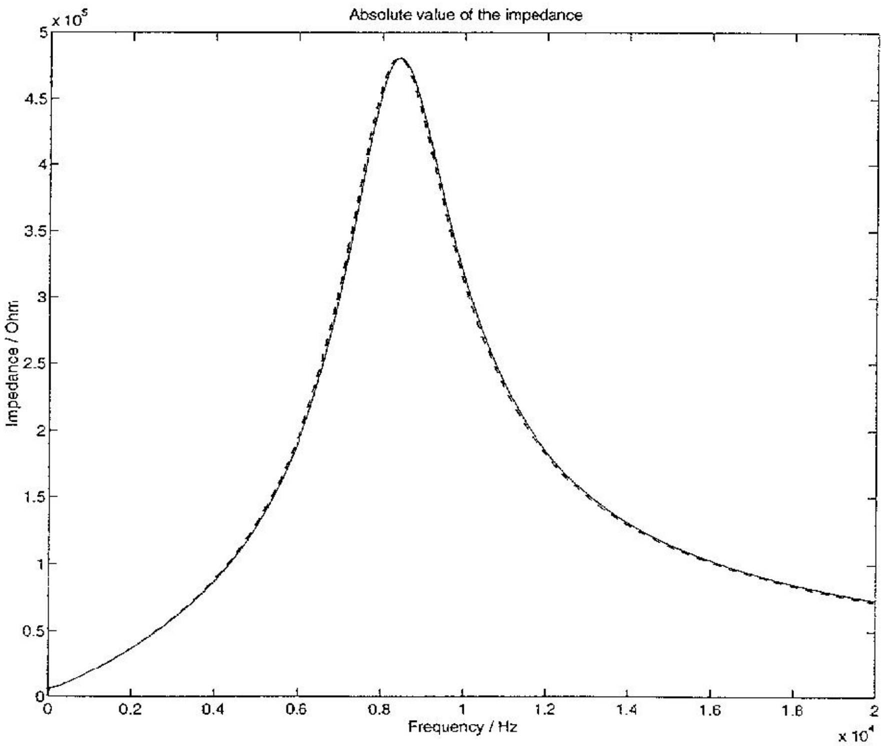
Stratocaster pick-up Nr. 2



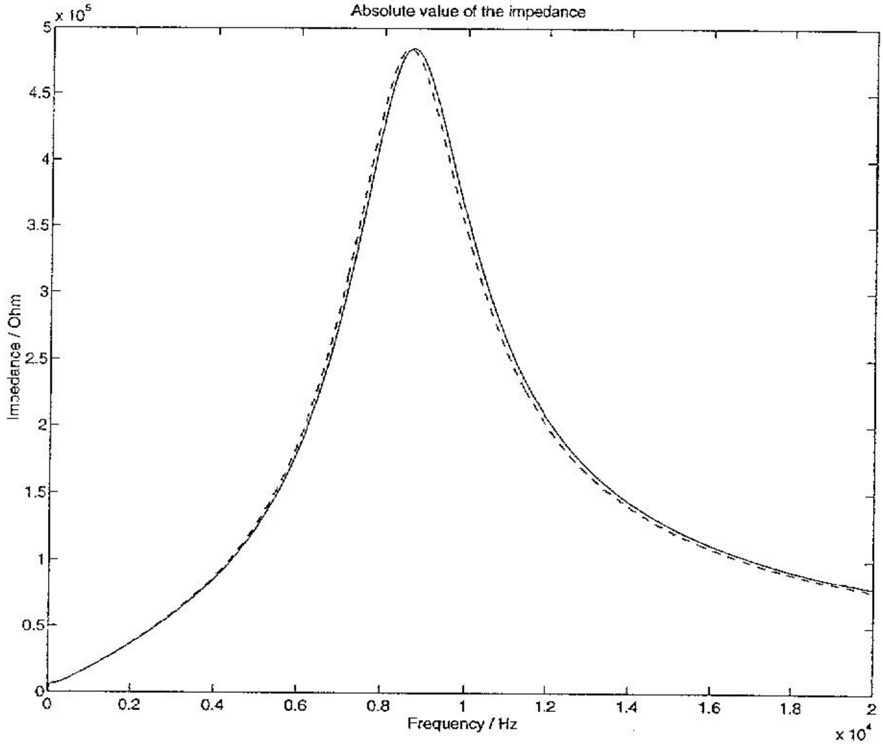
Stratocaster pick-up Nr. 3



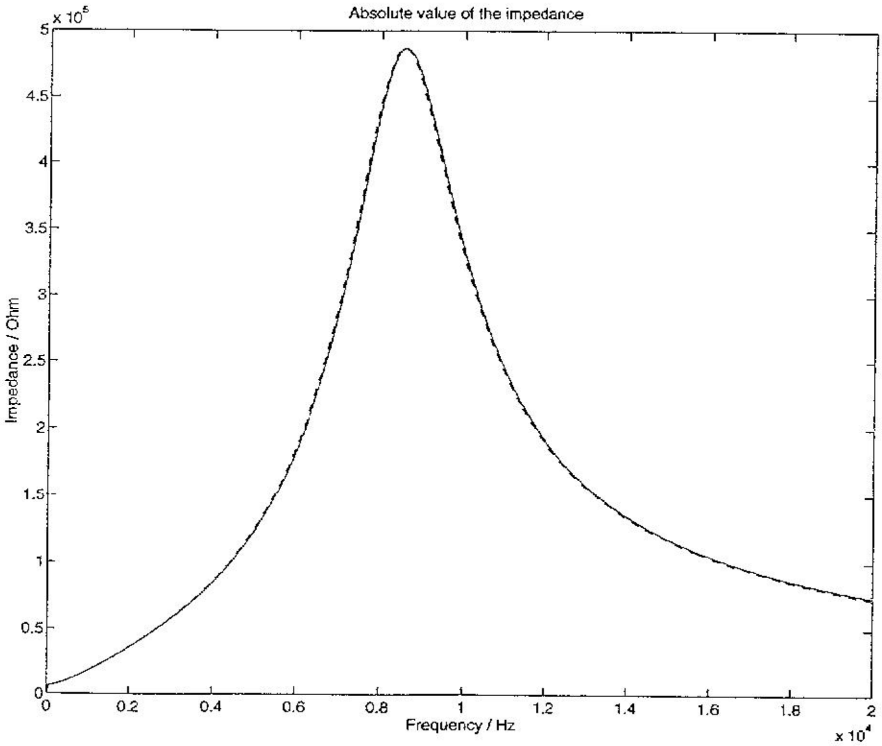
Stratocaster pick-up Nr 4



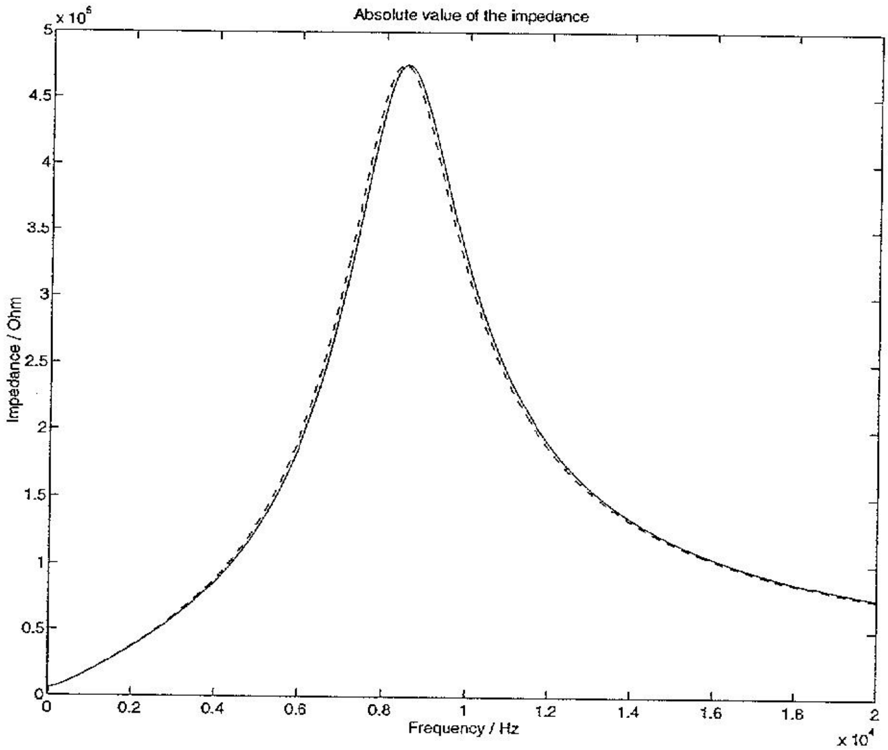
Stratocaster pick-up Nr. 5



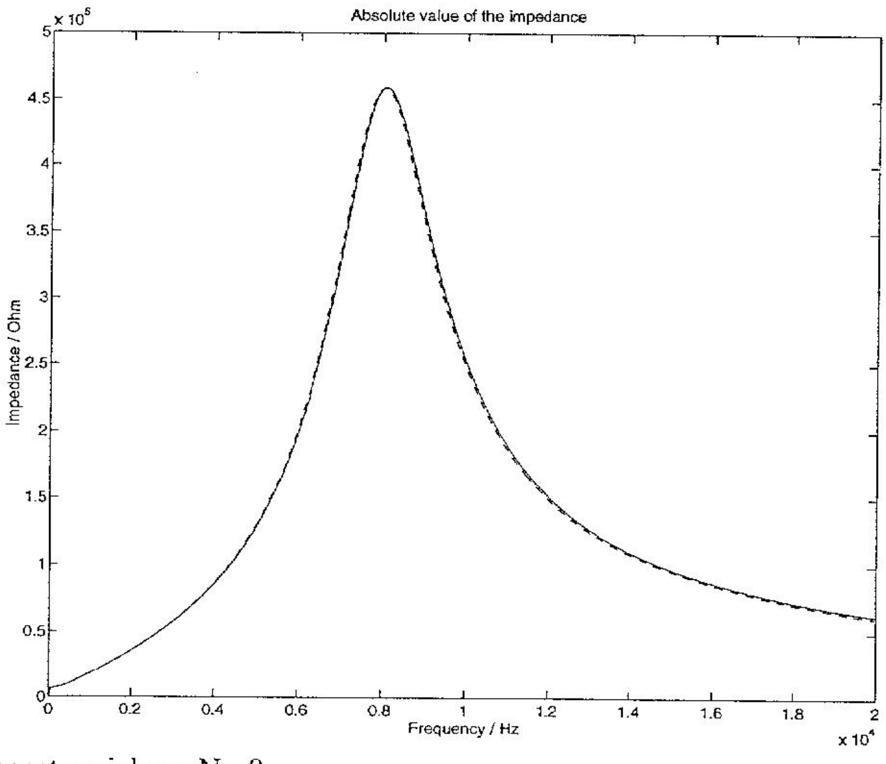
Stratocaster pick-up Nr 6



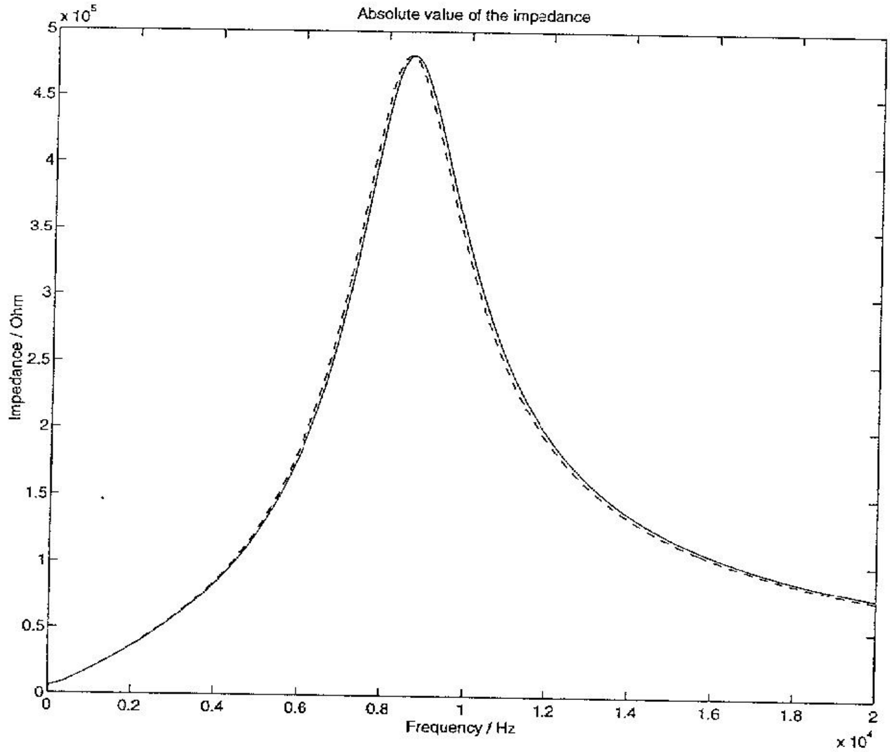
Stratocaster pick-up Nr. 7



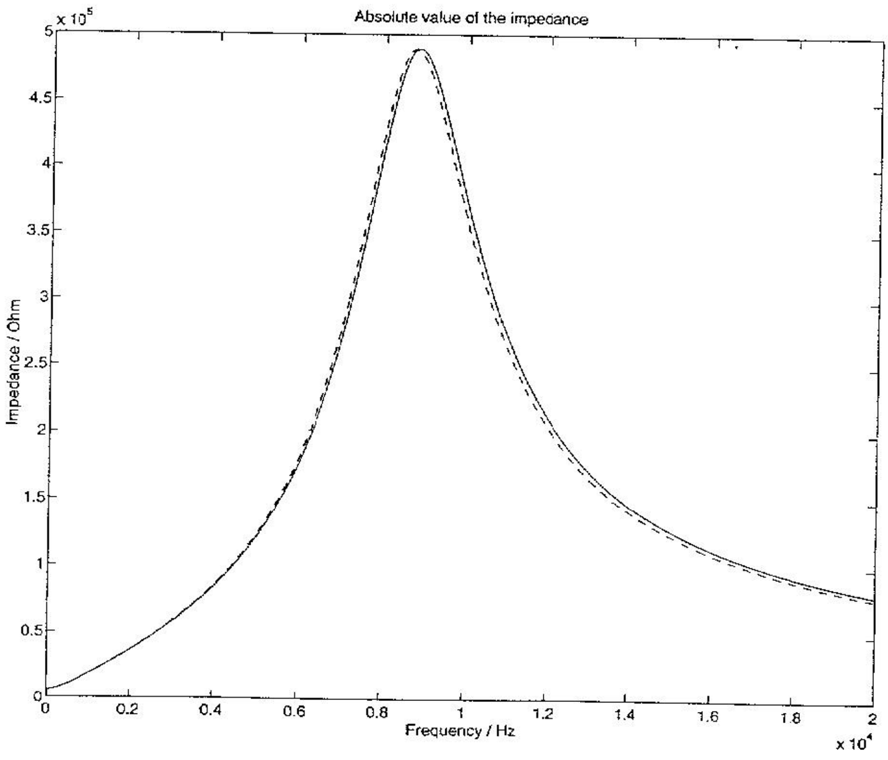
Stratocaster pick-up Nr 8



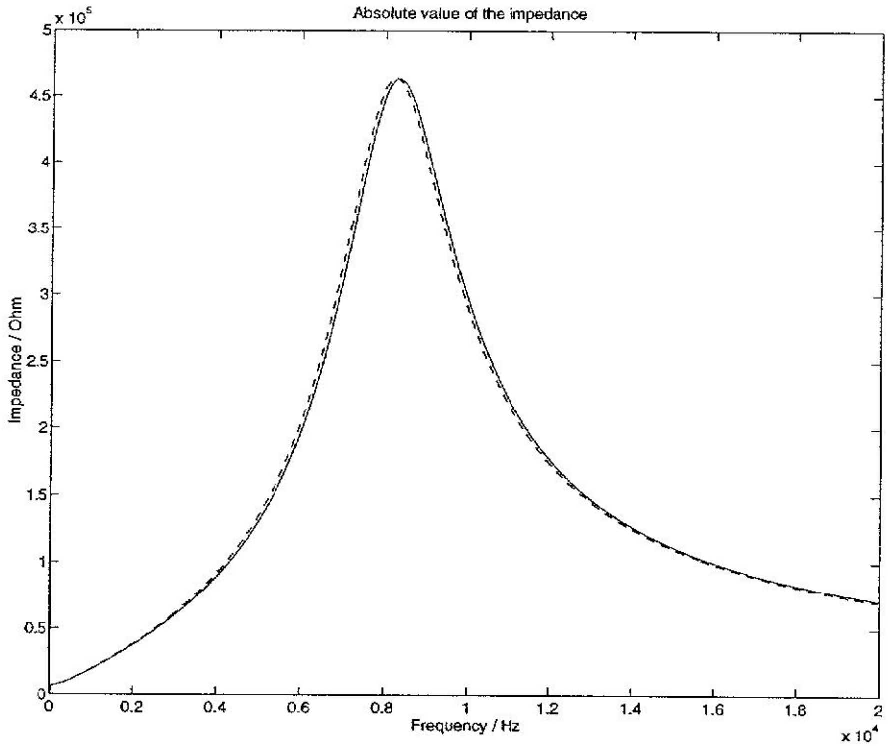
Stratocaster pick-up Nr. 9



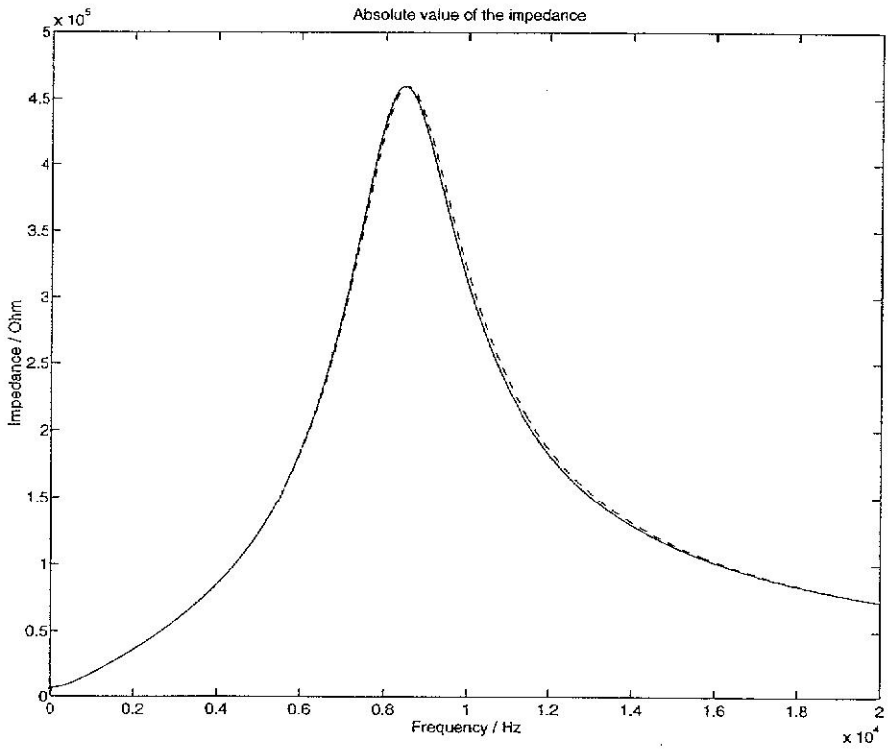
Stratocaster pick-up Nr 10



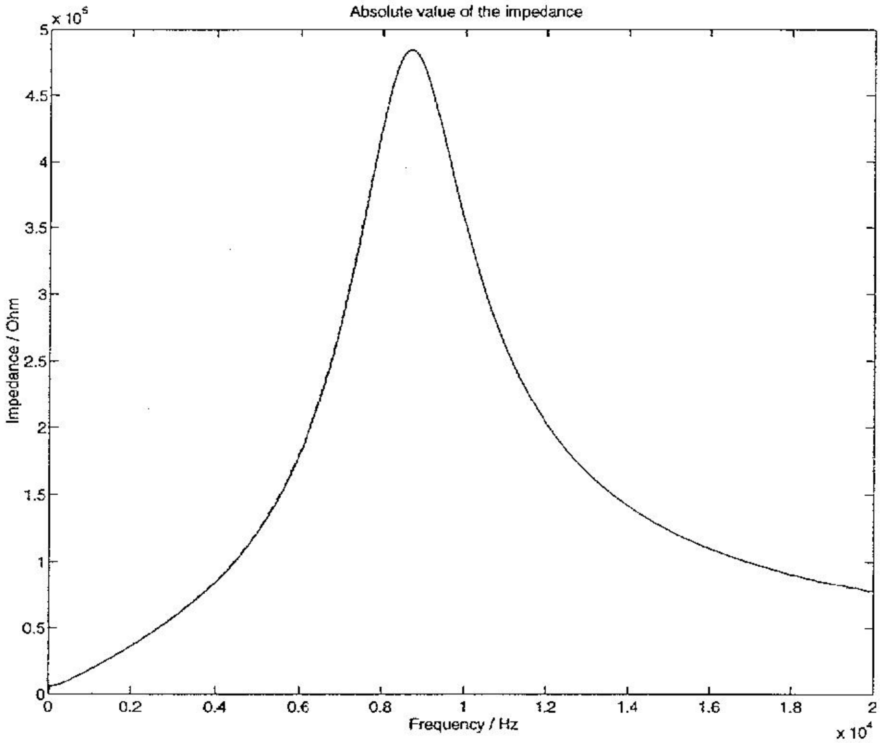
Stratocaster pick-up Nr. 11



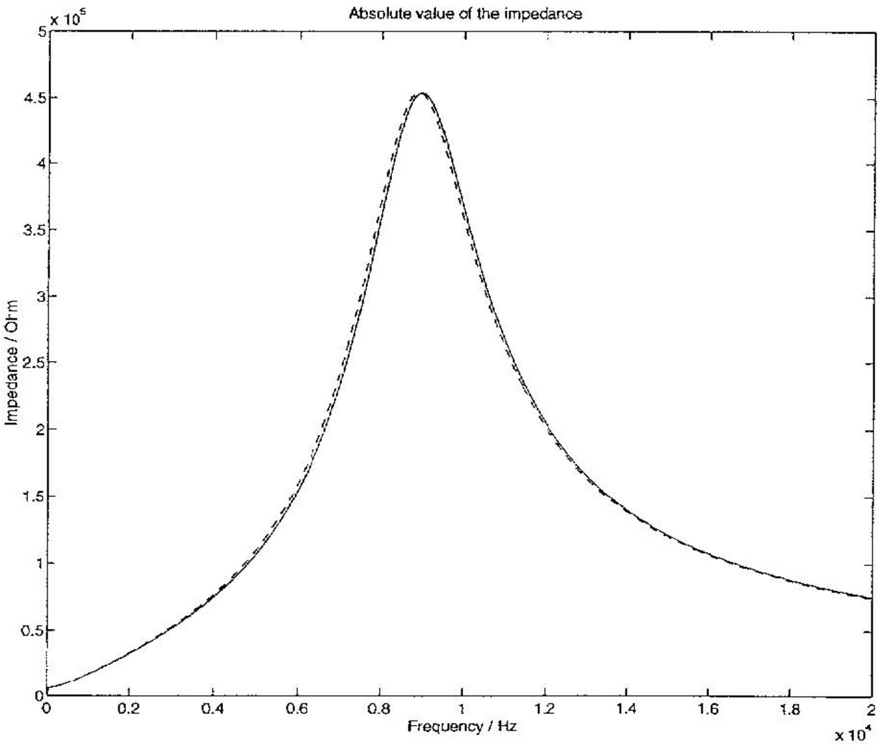
Stratocaster pick-up Nr 12



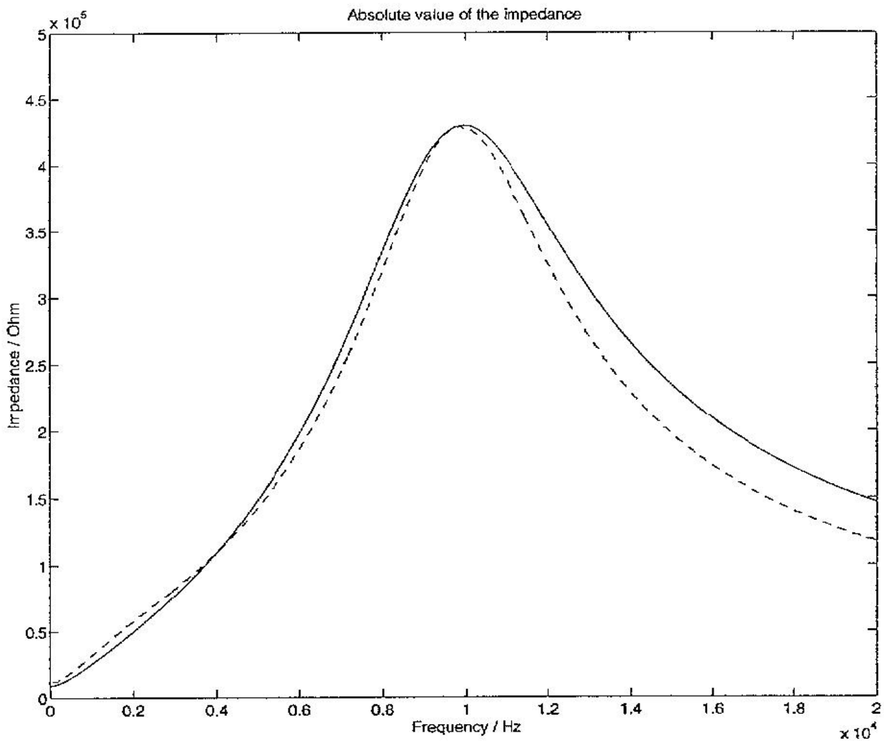
Stratocaster pick-up Nr. 13



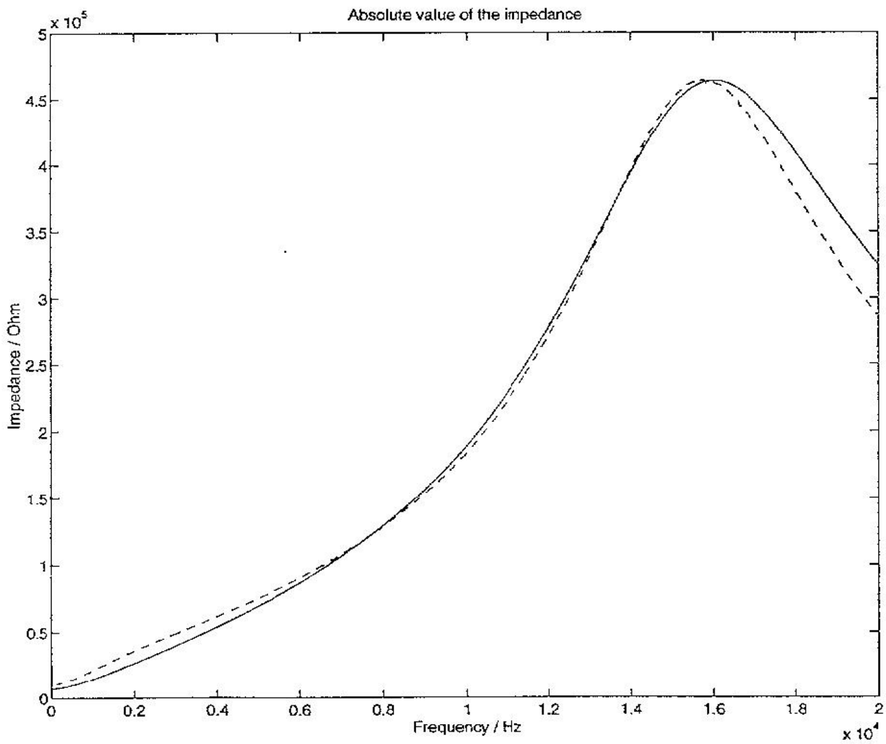
Stratocaster pick-up Nr 14



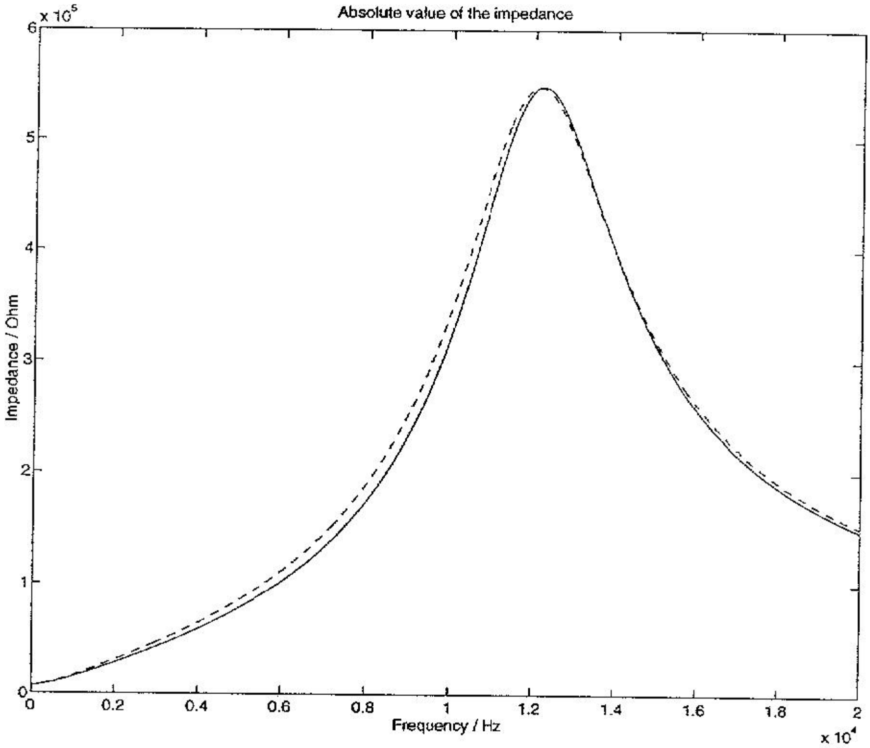
Stratocaster pick-up Nr. 15



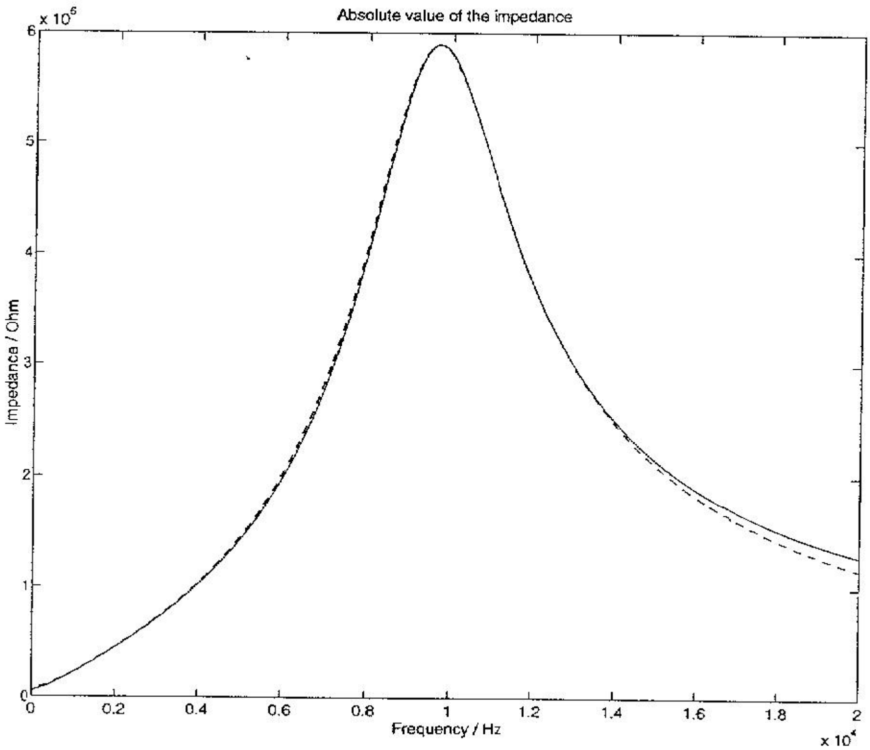
De Armond/Dynasonic 1957 Gretsch 1



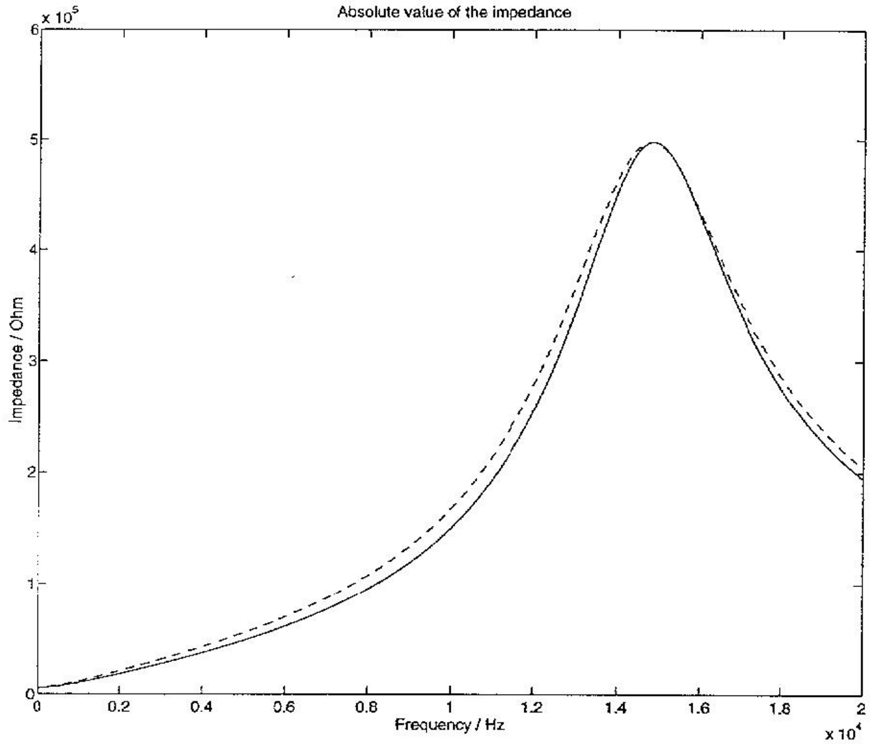
De Armond/Dynasonic 1957 Gretsch 2



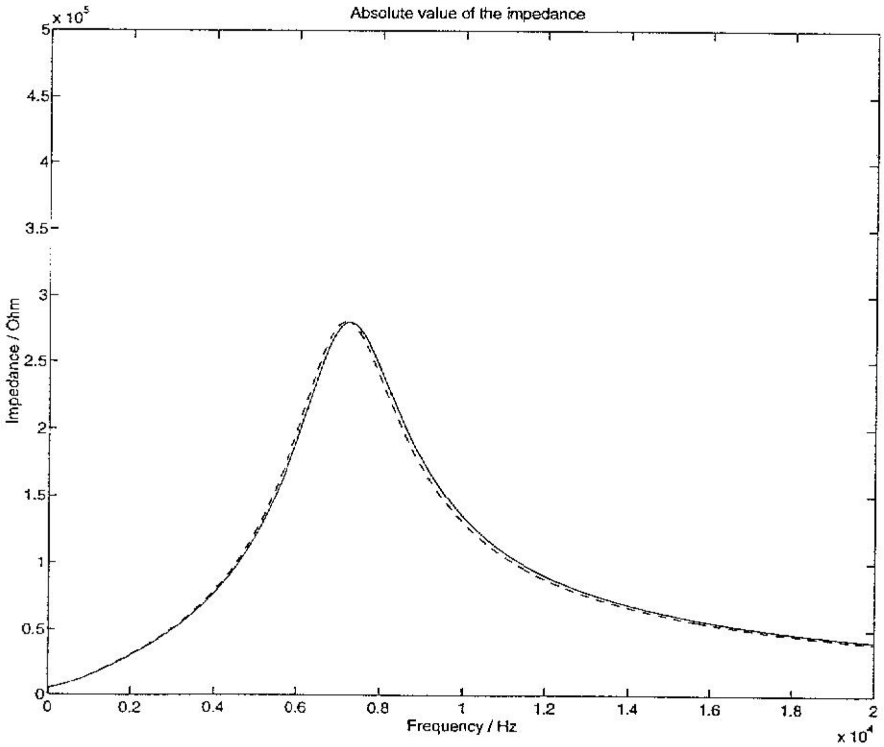
Voice pick-up proto 1



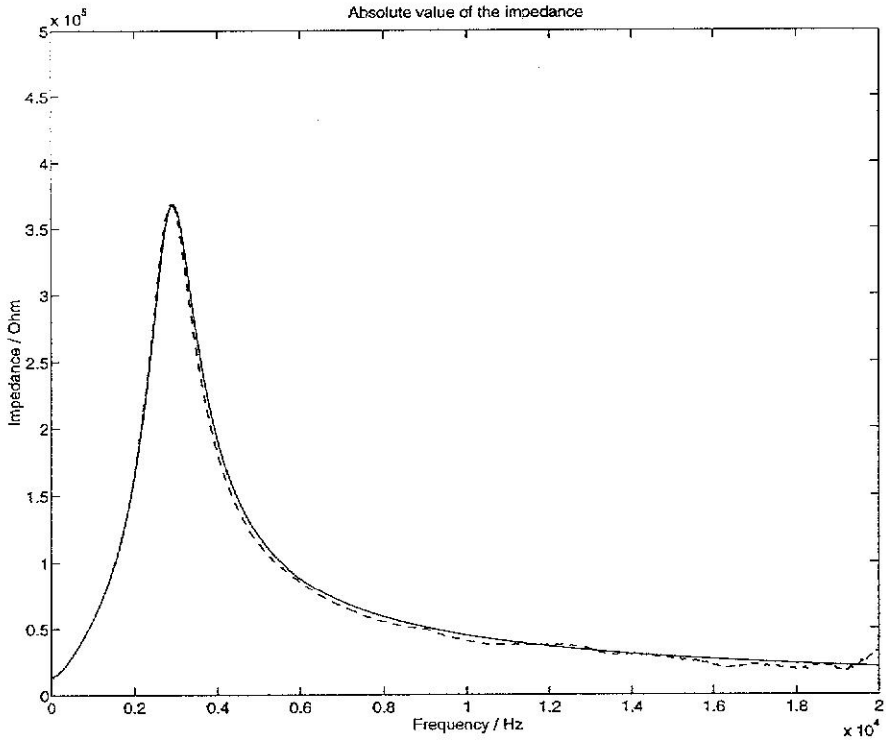
Voice pick-up proto 2



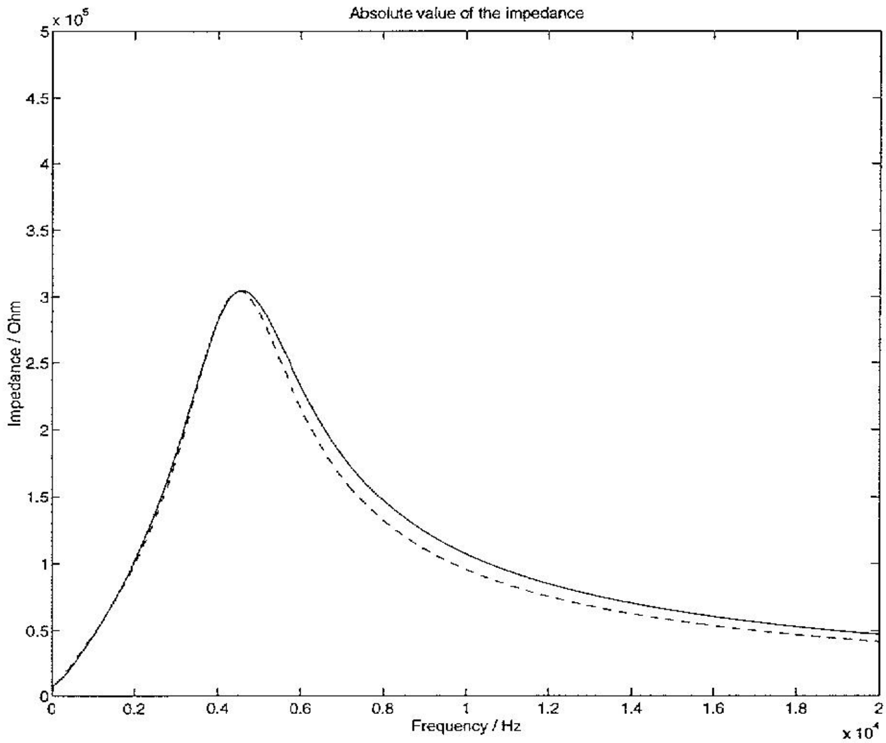
Voice pick-up proto 3



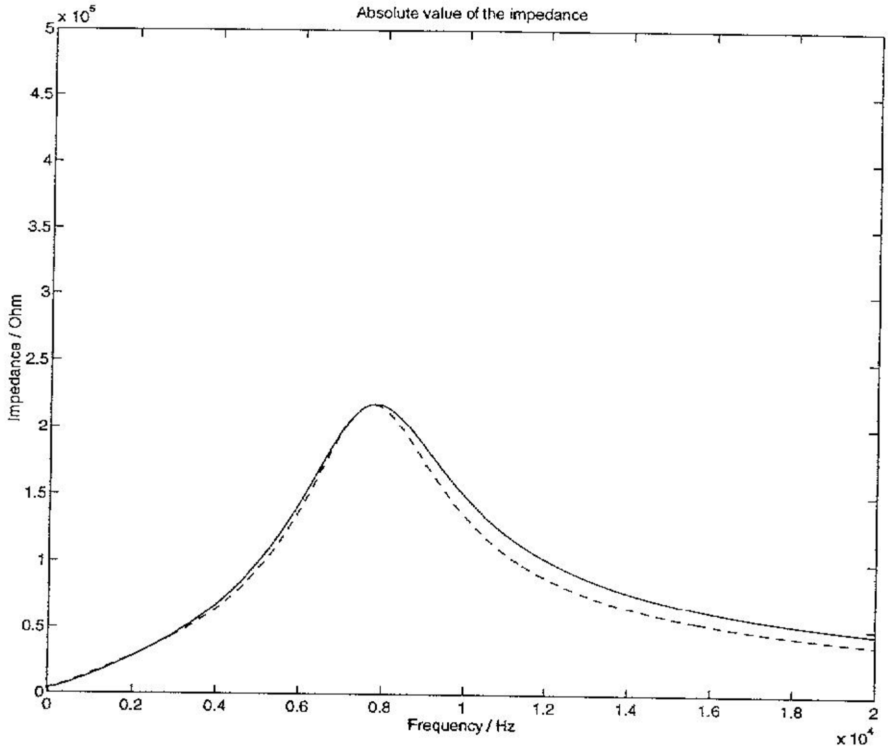
Fender Lace Sensor 1



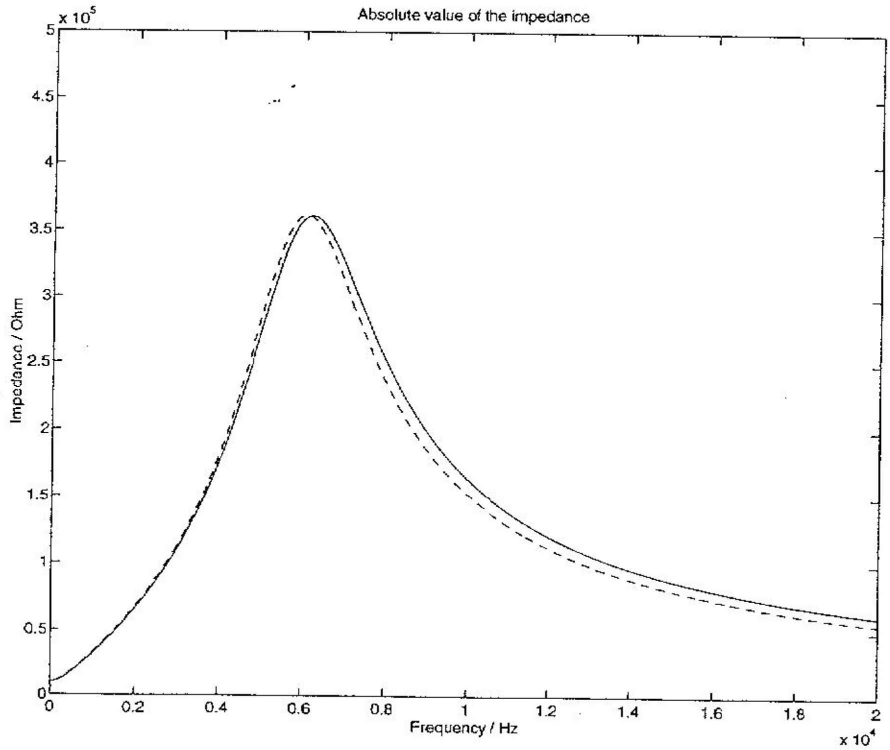
Fender Lace Sensor 2



Seymor Duncan P-90 Soapbar



Seymor Duncan Humbucker Jeff-Beck Model



Fender Humbucker

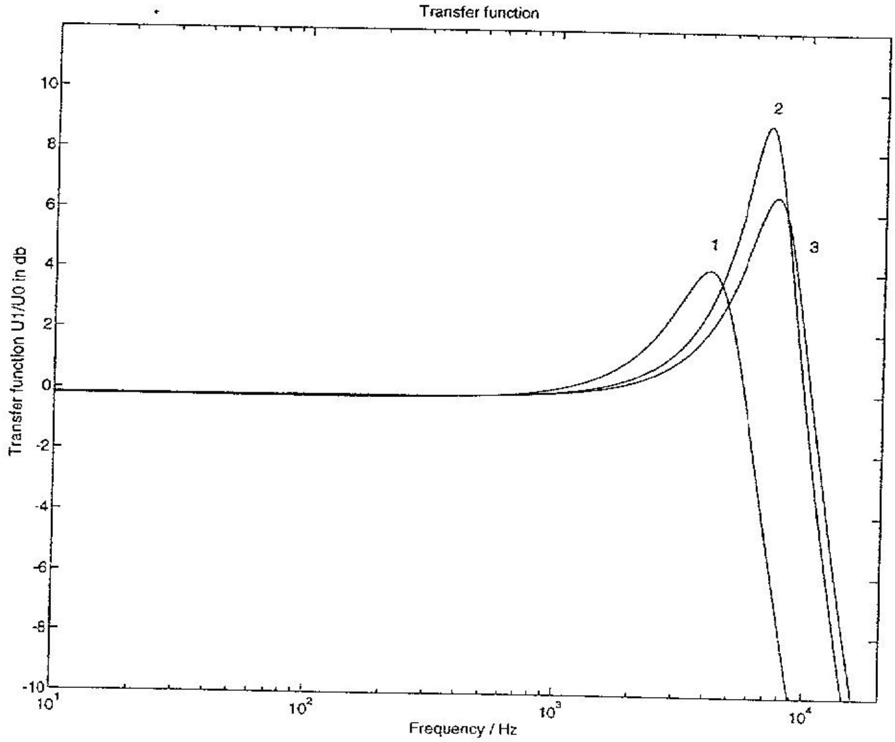
Theoretical transfer function of pick-ups

The figure below shows the transfer function of the following pick-ups:

1 Seymor Duncan P-90 Soapbar

2 Seymor Duncan Humbucker Jeff-beck Model

3 Fender Lace Sensor 1

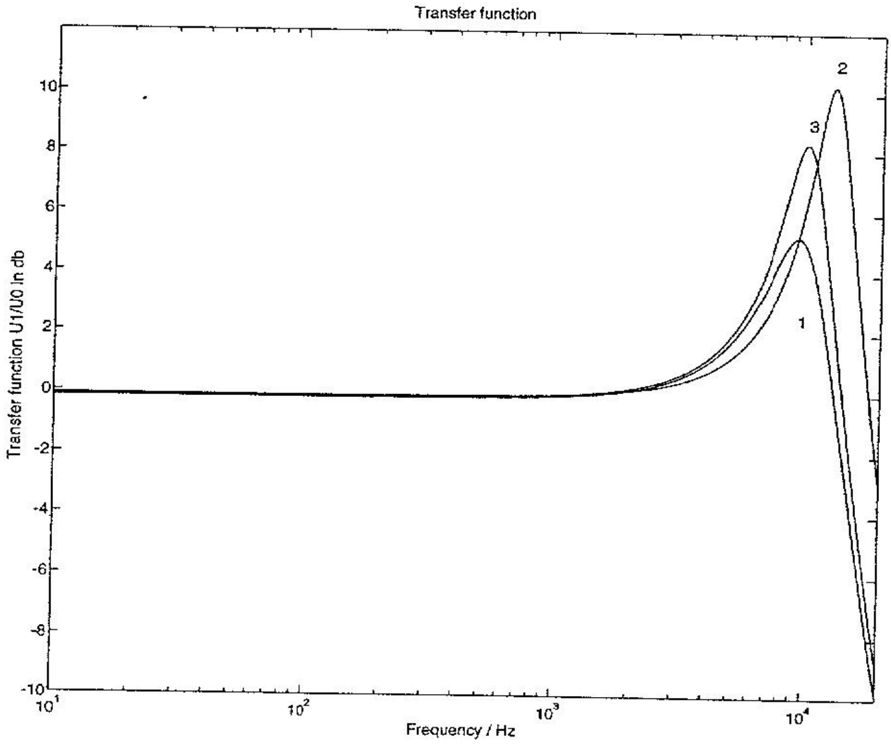


The figure below shows the transfer function of the following pick-ups:

1 De Armond/Dynasonic 1957 Gretsch 1

2 Voice pick-up proto 1

3 Voice pick-up proto 2



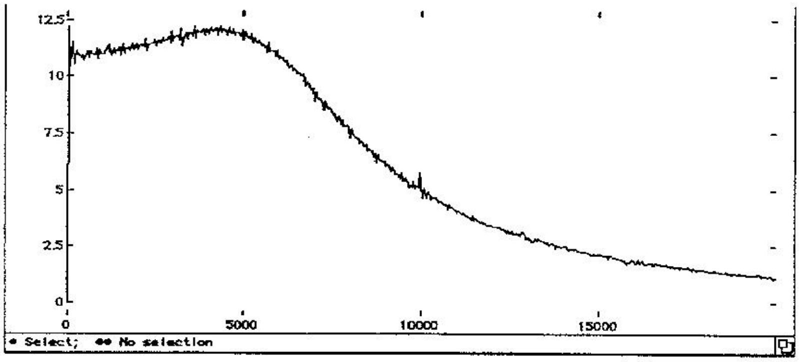
Measurements of section 3.8.3

The measurements were done with two different guitars:

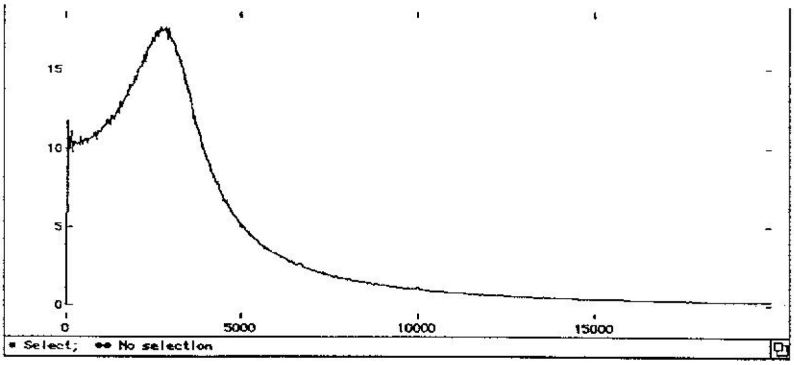
1) Stratocaster, Seymor Duncan.
2) Gibson Les Paul (Humbucker pick-ups).

Measurement conditions:

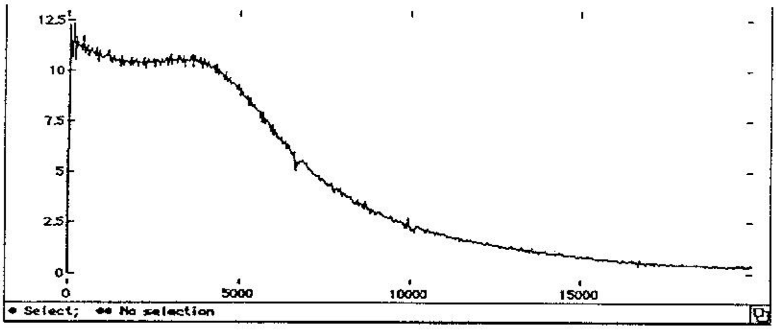
Maximum volume control.
Maximum tone control.
x-axis: Frequency range from 0 Hz to 20 kHz
y-axis: Relative transfer function U_1/U_o



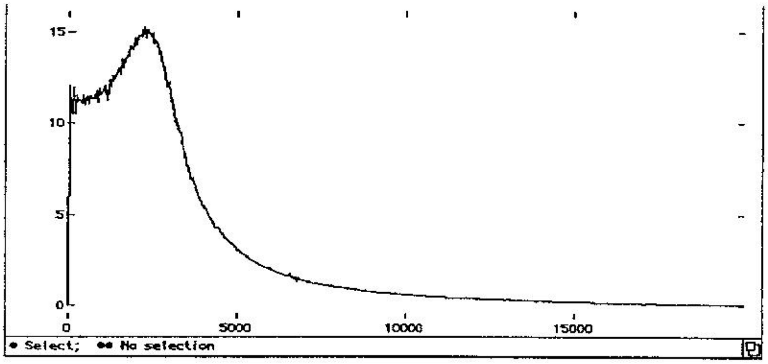
Stratocaster front pick-up, no capacitance.



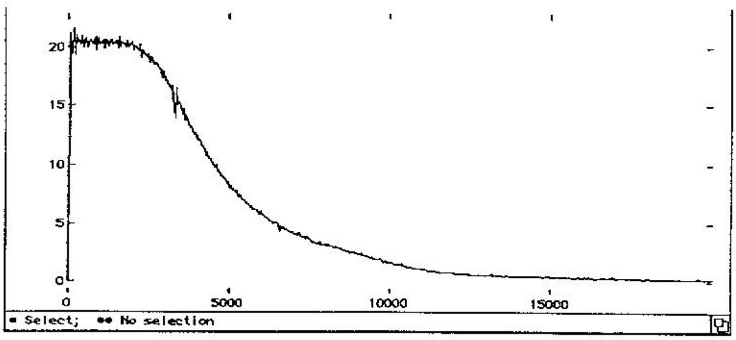
Stratocaster front pick-up, load of 820 pF.



Gibson les Paul front pick-up, no capacitance.



Gibson Les Paul front pick-up, load of 820 pF.



Gibson Les Paul bridge pick-up, no capacitance.

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