Physical modeling of a nonlinear vibrating string

Jyri Pakarinen 27.5.2004

Helsinki University of Technology

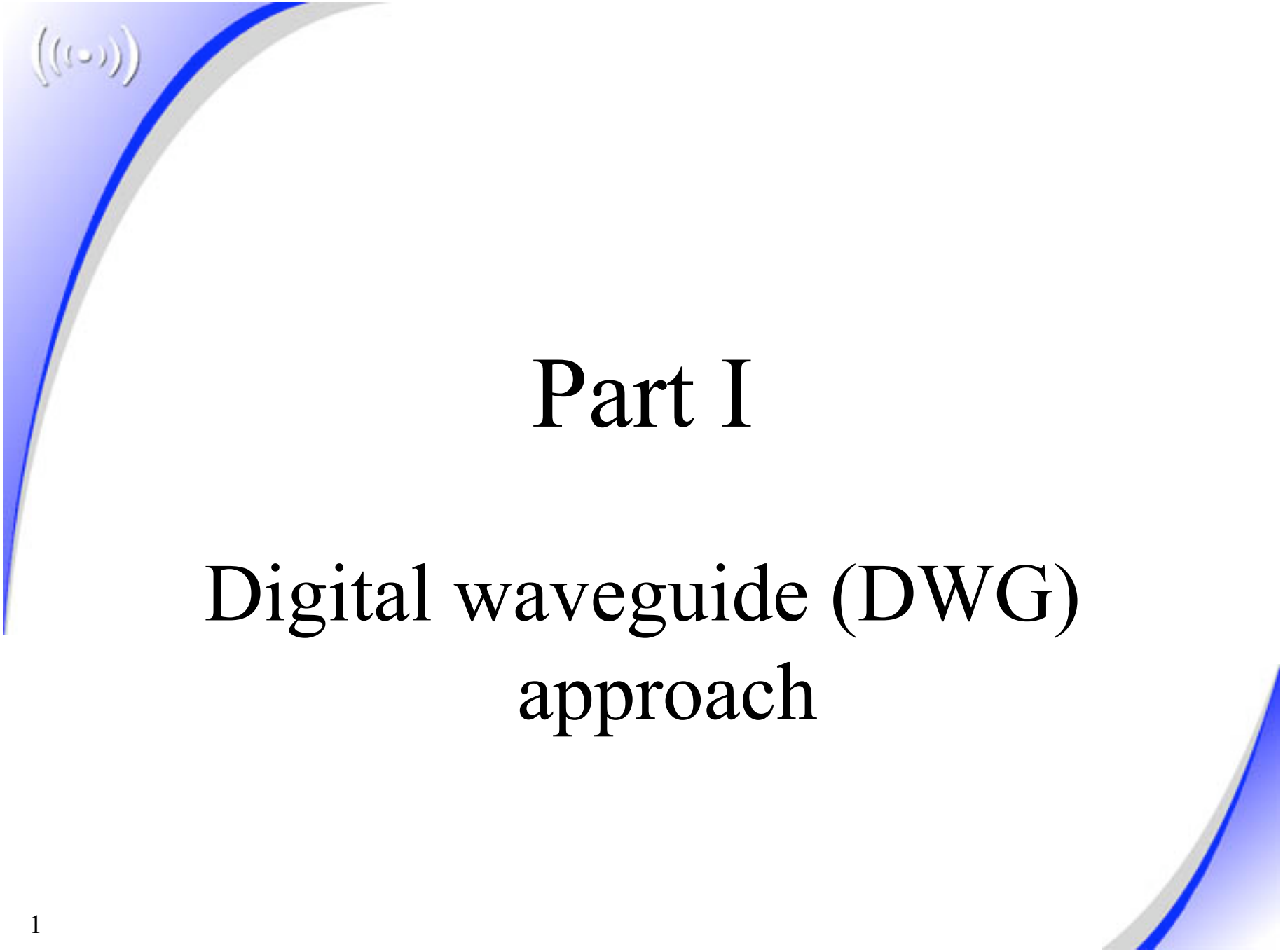
Laboratory of Acoustics and Audio Signal Processing

Contents

- The problem: modeling nonlinear strings
- Part I: Digital waveguide (DWG) approach
- Construction of the kantele string model
- Part II: Finite difference time-domain (FDTD) approach
- Sound examples
- Conclusions

Problem: Physical modeling of a nonlinear vibrating string

- Real string vibration elastic (nonlinear)
- Tension modulation results in
  - Initial pitch glides
  - Generation of missing harmonics
- We want to construct a computational model which simulates this behavior



Part I

Digital waveguide (DWG) approach

Linear string model: Theory

- Vibration of a string can be modeled by the 1-D wave equation:

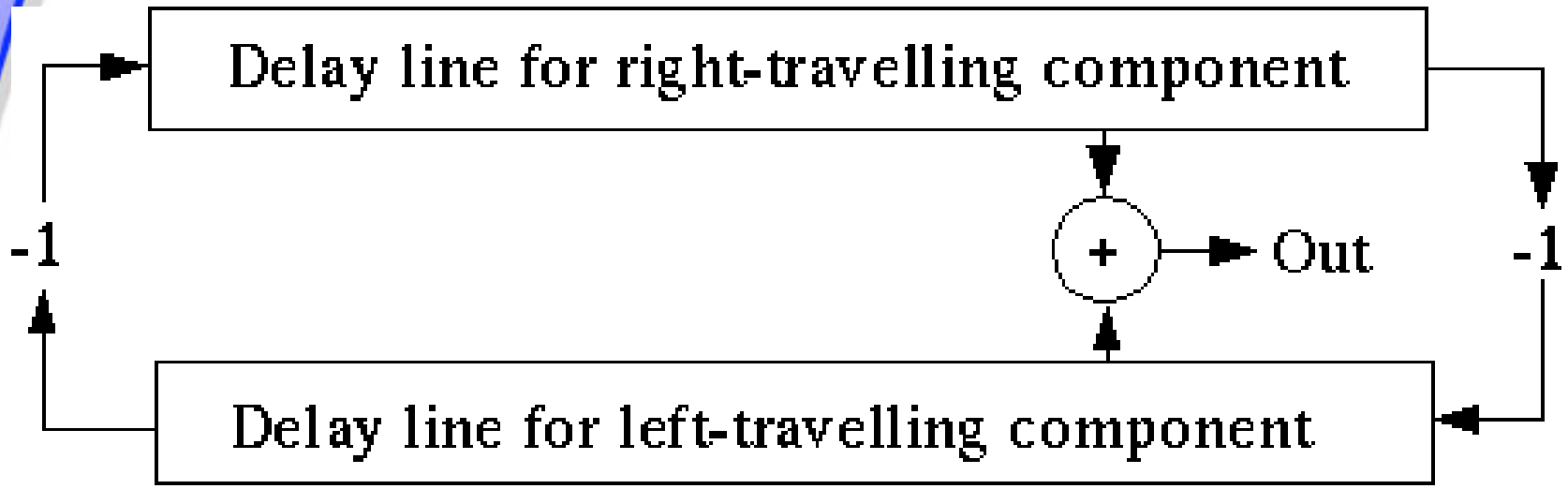
$$K\frac{(\partial^2 y)}{(\partial x^2)} = \epsilon\frac{(\partial^2 y)}{(\partial t^2)}$$

- Traveling wave solution:

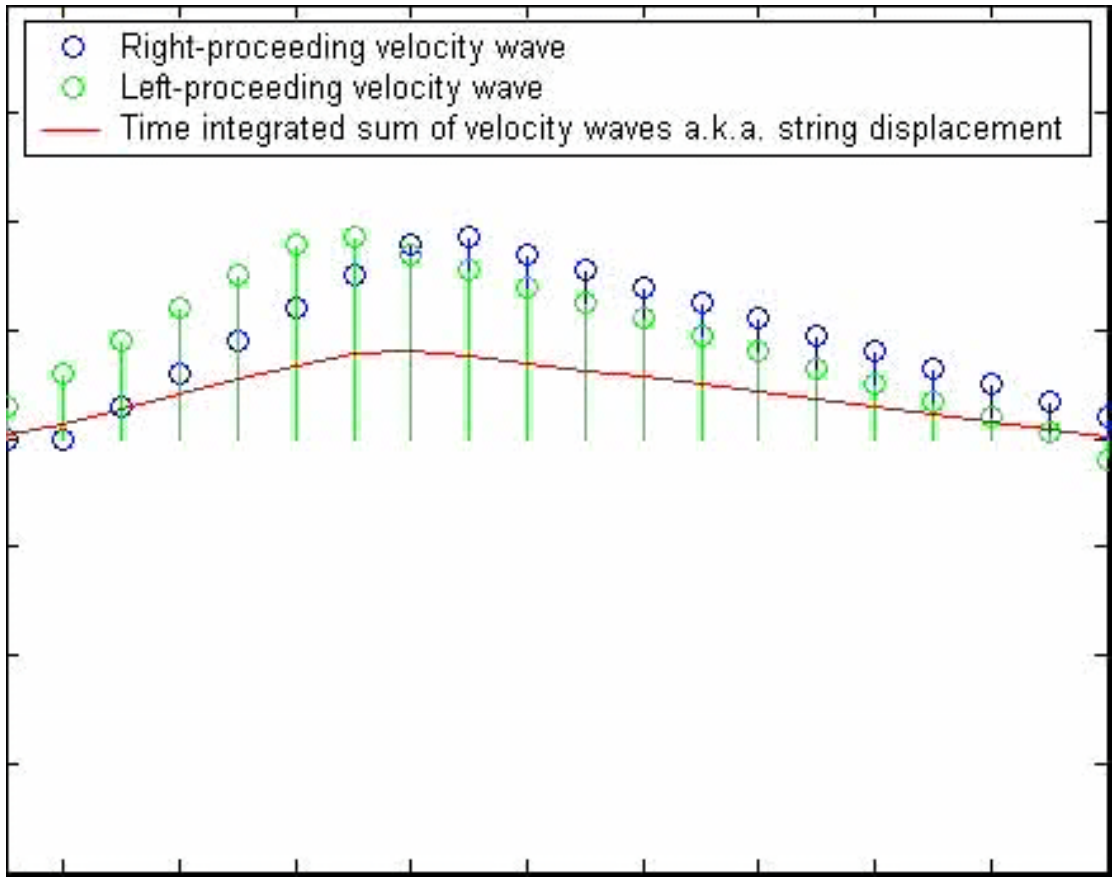
$$y(x,t) = y_r(t\text{-}x/c) + y_l(t\text{-}x/c)$$

- Can be implemented as two delay lines!

Linear string model: DWG

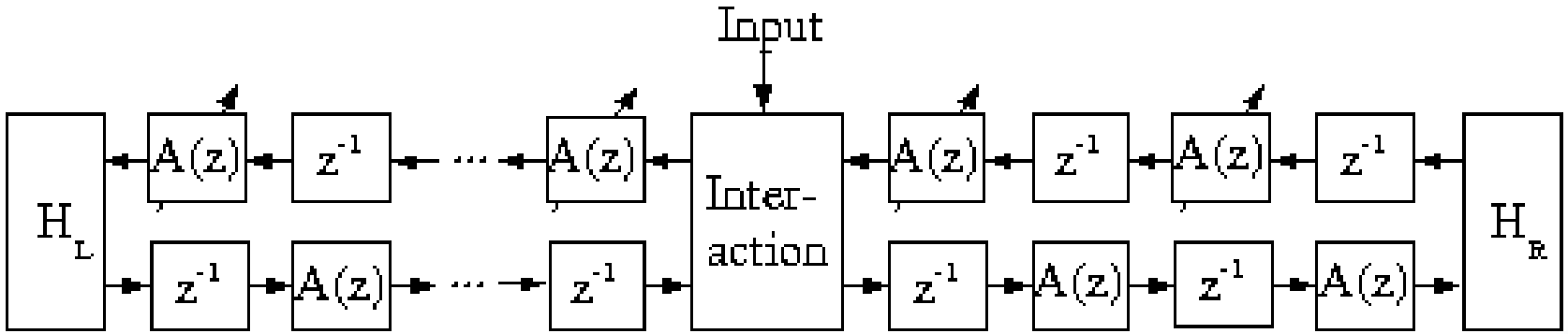


Linear string model: DWG example



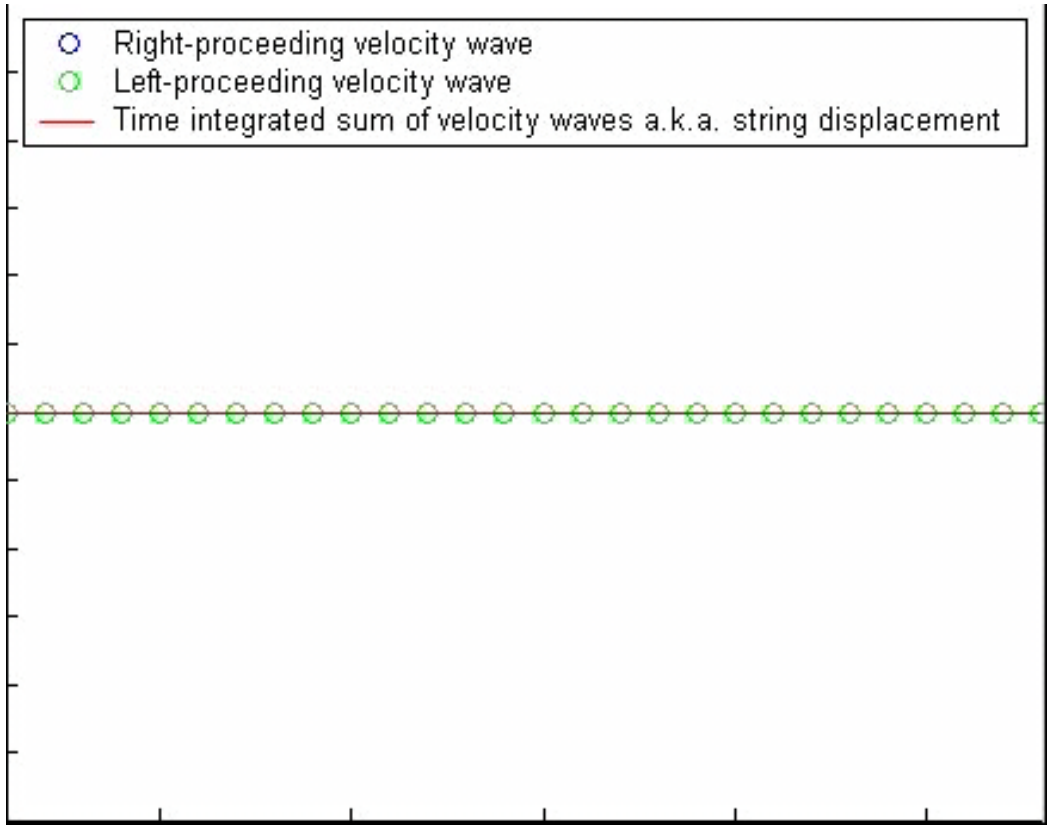
Nonlinear DWG string model

- Time-varying allpass filters distributed along the string in order to modulate delay line lengths



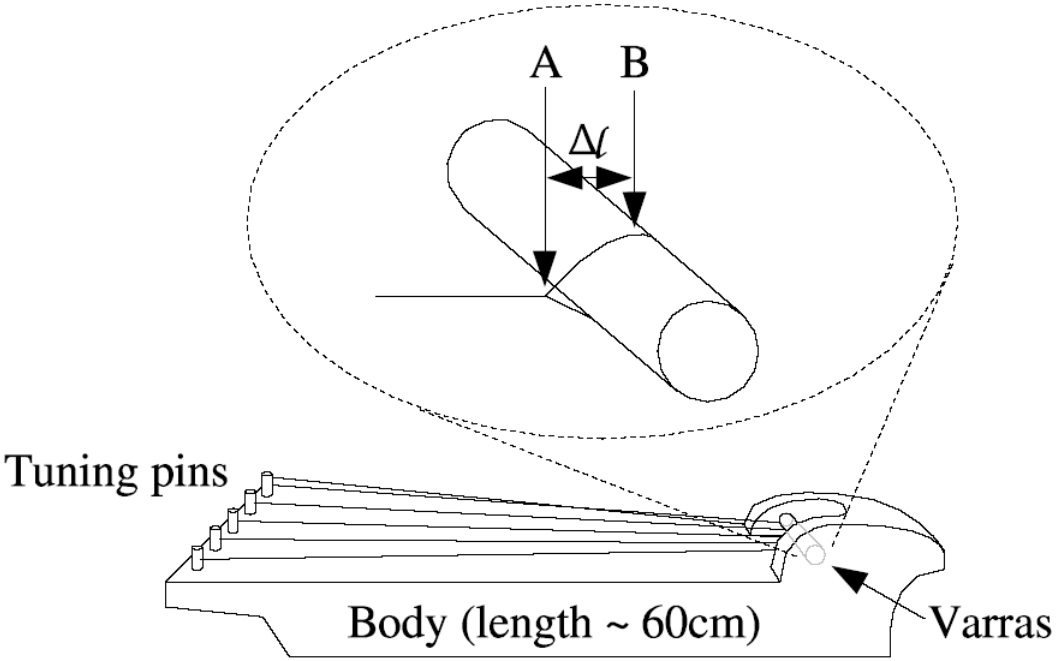
Nonlinear DWG string model (cont.)

- Interaction possible during run-time



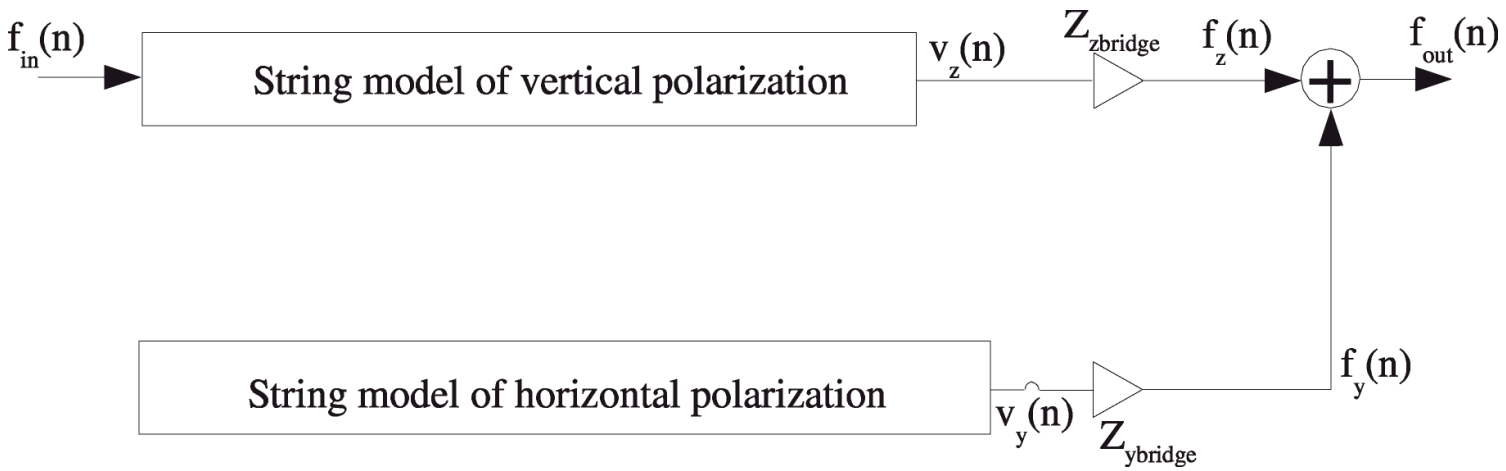
Kantele string model

- Vibrational polarizations have different lengths



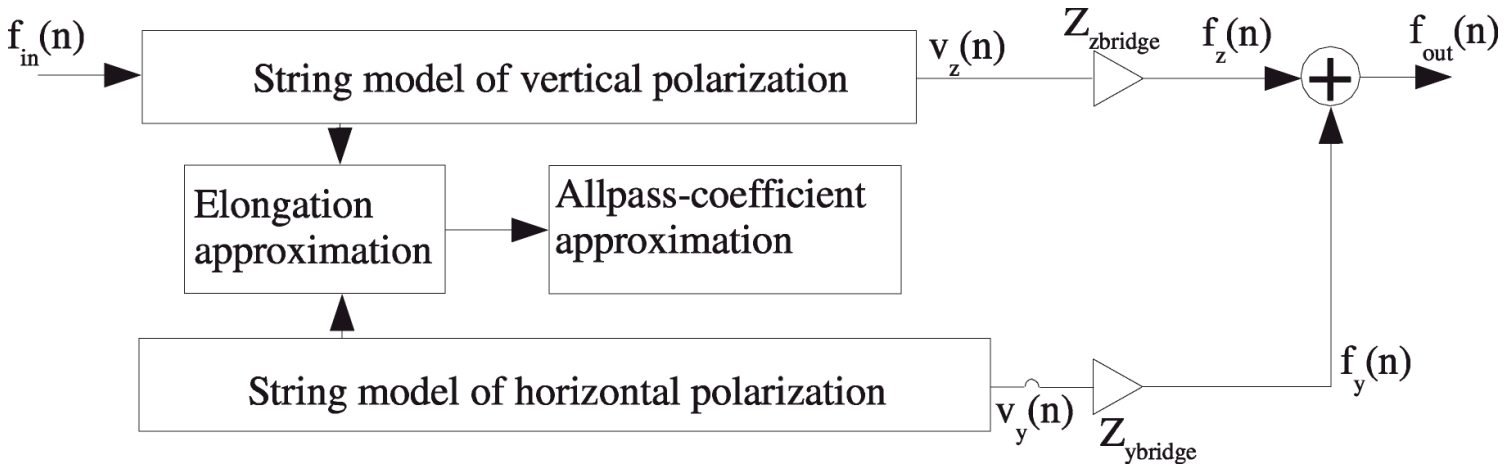
Kantele string model (cont.)

- Uses two nonlinear DWG models with slightly different lengths



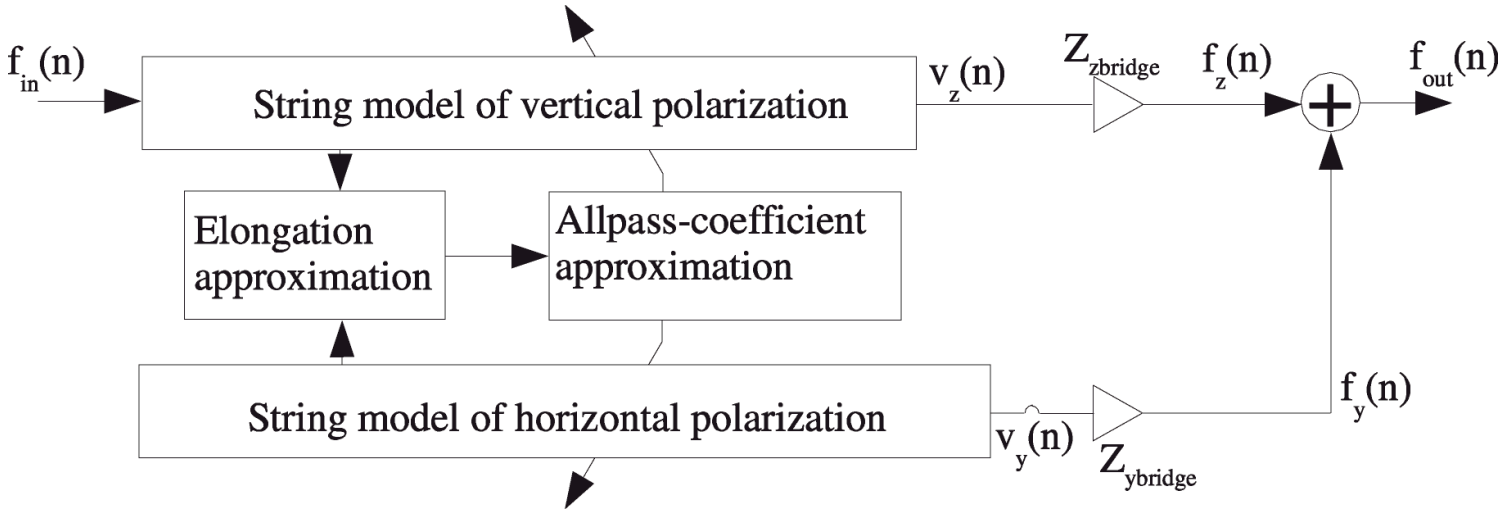
Kantele string model (cont.)

- Uses two nonlinear DWG models with slightly different lengths



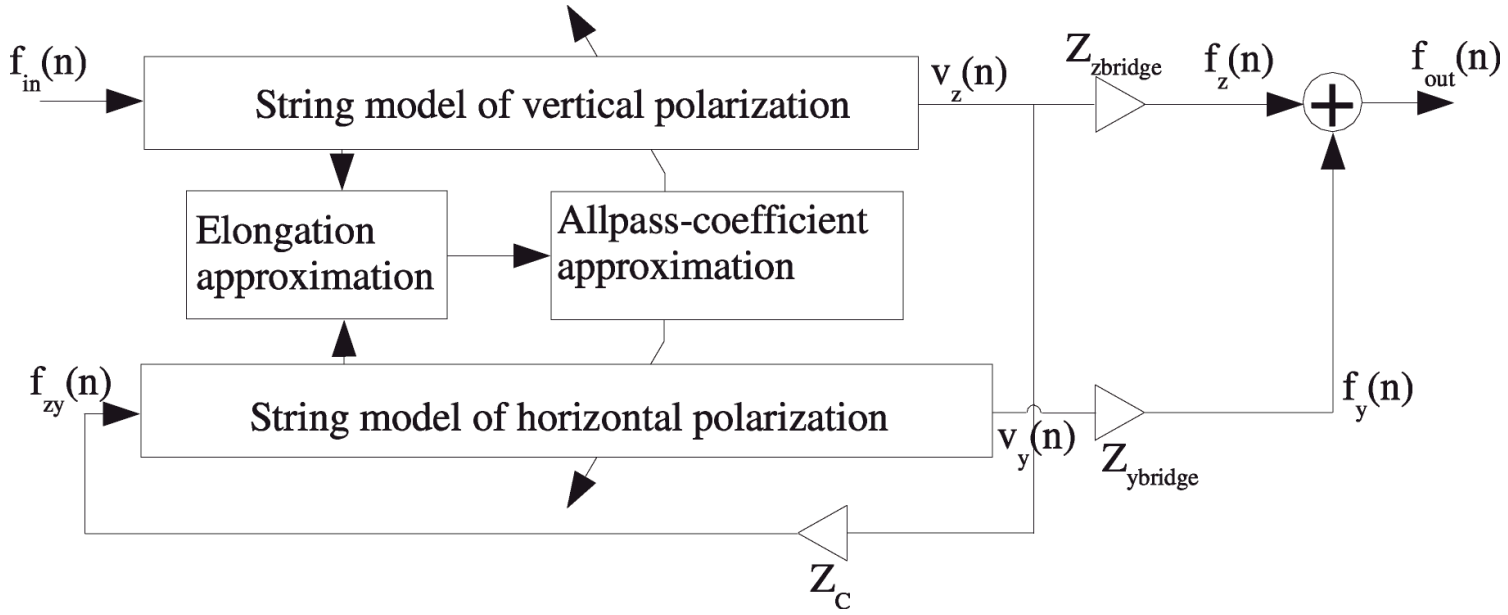
Kantele string model (cont.)

- Uses two nonlinear DWG models with slightly different lengths



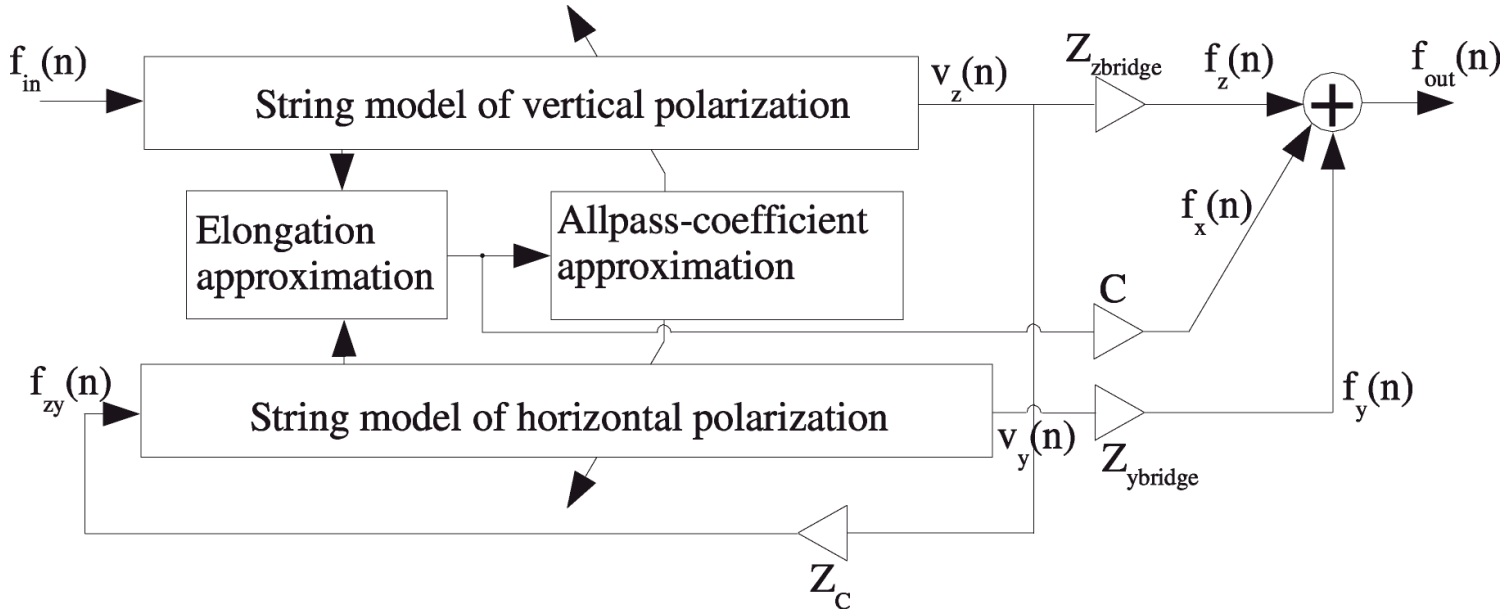
Kantele string model (cont.)

- Uses two nonlinear DWG models with slightly different lengths



Kantele string model (cont.)

- Uses two nonlinear DWG models with slightly different lengths

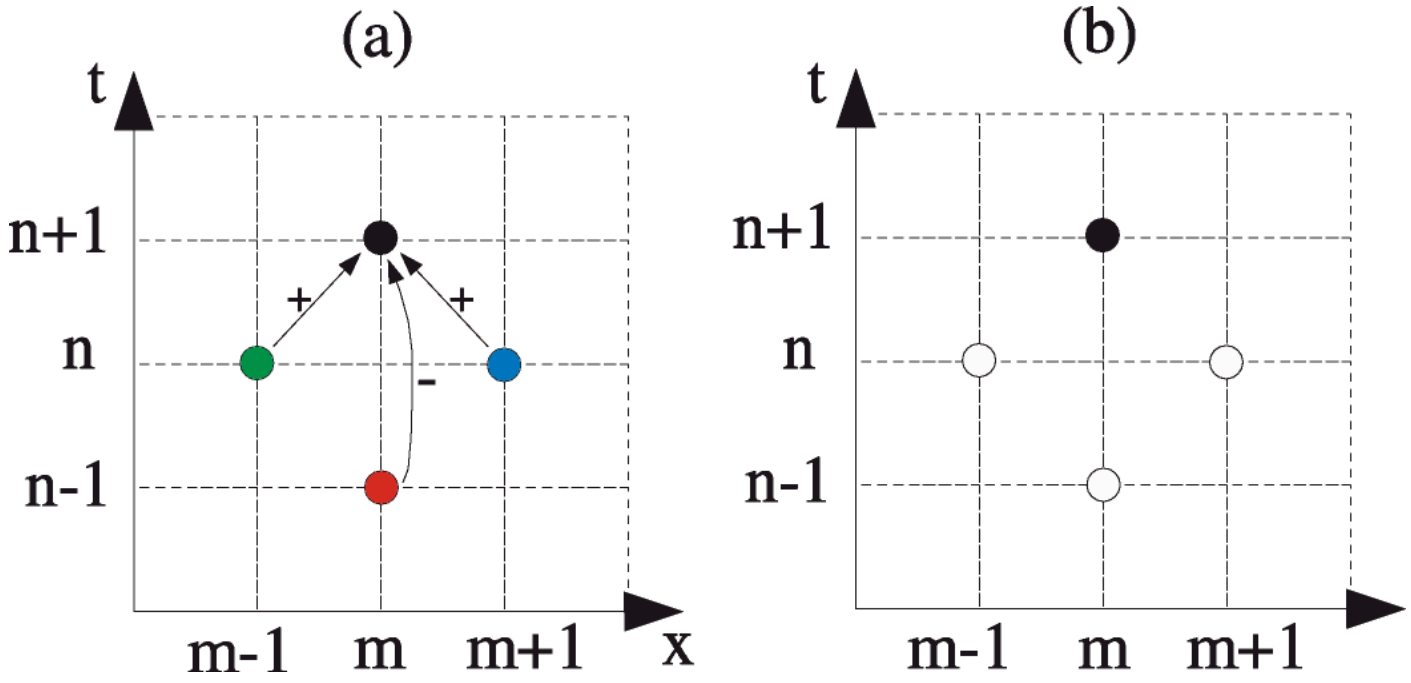


Part II

Finite difference time-domain (FDTD) approach

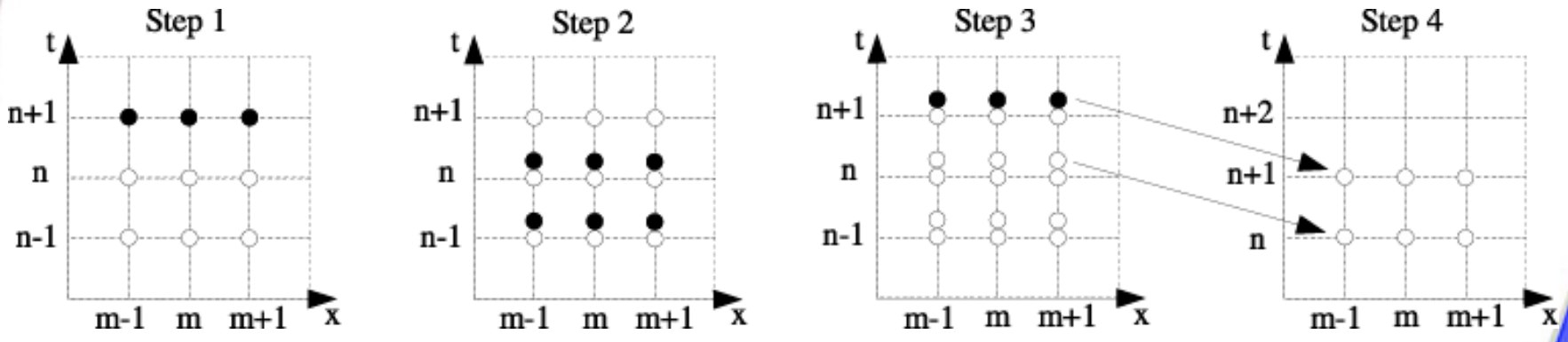
Linear FDTD string

- Derivatives approximated with differences in the wave equation
- **y(n+1,m) = y(n,m-1) + y(n,m+1) - y(n-2,m)**



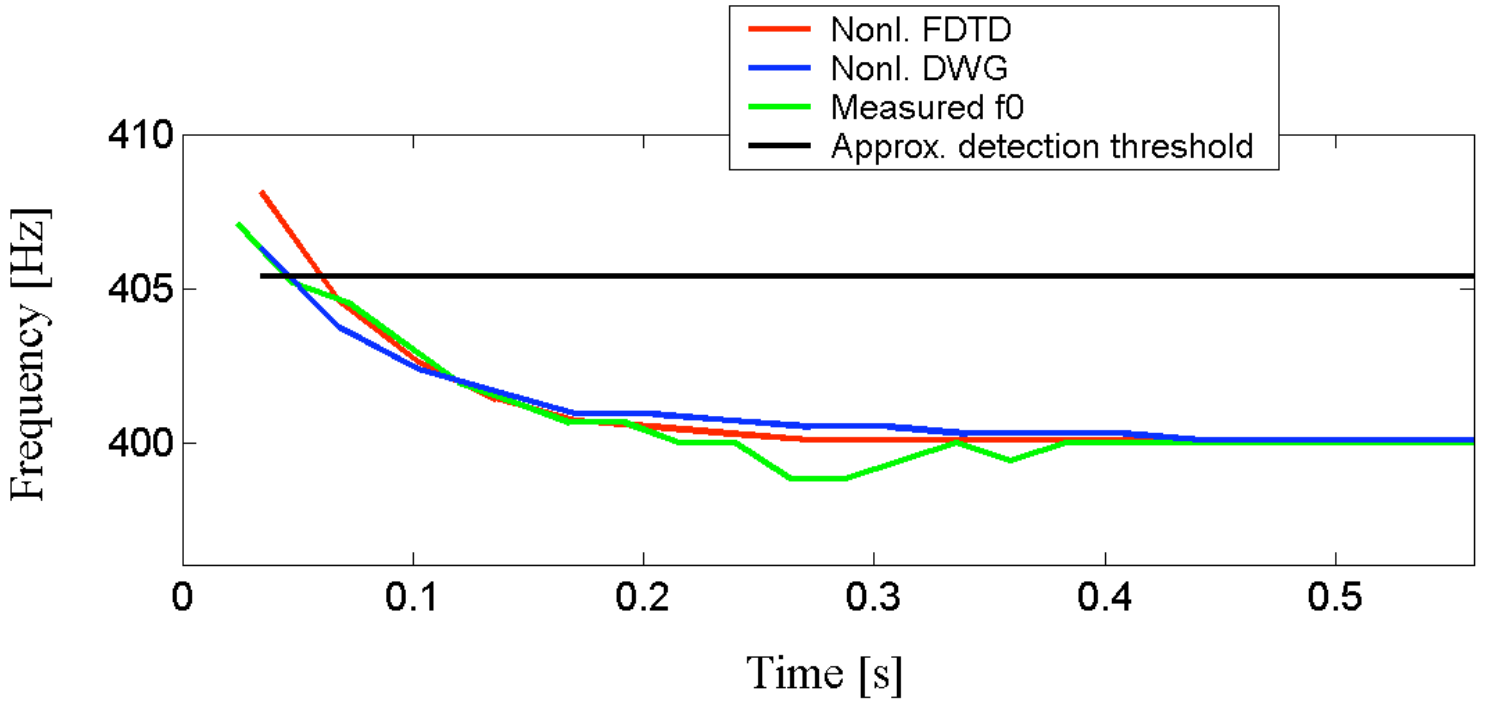
Nonlinear FDTD string

- Processing speed of the linear FDTD string can be modulated by interpolating in time



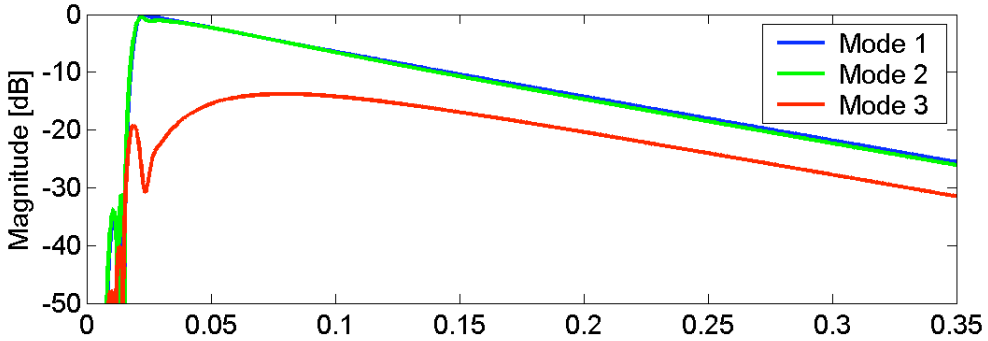
Synthesis results

- Fundamental frequency glide modeled realistically by both models

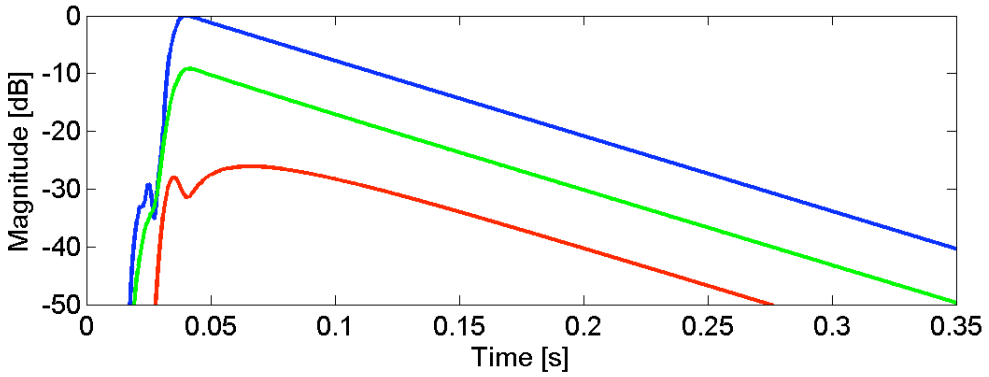


Synthesis results

- The DWG approach synthesizes better the generation of missing harmonics
- DWG:



- FDTD:



Sound examples

- Nonlinear DWG string was plucked at 1/3 of its length
- Allpass-coefficient scaling was varied
- A = 0 (linear string) 
- A = 0, 0.2, 0.4, … 1 
- A = 5 (exaggerated nonlinearity) 

Conclusions

- Two physical models for simulating nonlinear strings were presented
- In the DWG approach, **time varying allpass filters** were used in modulating the delay line lengths
- In the FDTD approach, **interpolation between time samples** was used in modulating the time step of the algorithm

Questions?